UCF
Calc III

– EXAM 1 NOTES

The following topics are covered:

- ☐ Dot Product & Basic Properties
- ☐ Angle Between Vectors
- ☐ Scaler & Vector Projections
- ☐ Cross Product & Properties
- ☐ Area of a Parallelogram
- ☐ Equation of a Line
- ☐ Intersection of Planes
- ☐ Equation of a Plane
- ☐ Parallel, Skew, & Intersecting
- ☐ Limits of Vector Functions
- ☐ Calculus of Vector Functions
- ☐ Arc Length
- ☐ Curvature
- ☐ TNB-Frames
- ☐ Osculating Plane
- ☐ Limits approaching via Axes
- ☐ Partial Derivatives

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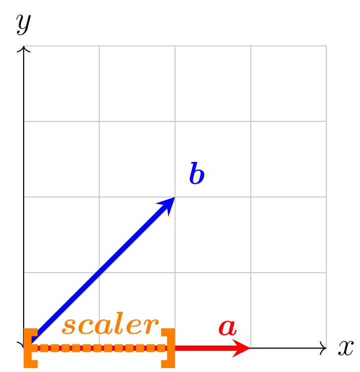
Dot Product formula: $\vec{a} \cdot \vec{b} = a_x b_x + a_y n_y + a_z b_z$

Properties of Dot Product:

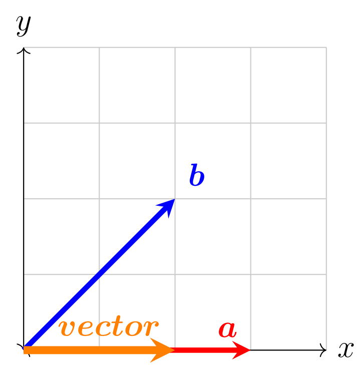
- $if\ \vec{a} \cdot \vec{b} = 0,\ then\ \vec{a}\ is\ perpendicular\ to\ \vec{b}$
- $\vec{a} \cdot \vec{a} = ||\vec{a}||^2$
- $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$
- $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$
- $k\vec{a} \cdot \vec{b} = k(\vec{a} \cdot \vec{b})$
- $\vec{0} \cdot \vec{a} = 0$

Angle between Vectors: $\vec{a} \cdot \vec{b} = ||\vec{a}||||\vec{b}|| \cos\theta$

Scaler Projection: The magnitude of the $\vec{b}_a$ component.
$comp_{\ \vec{a}}\ \vec{b} = \frac{\vec{a} \cdot \vec{b}}{||\vec{a}||}$



Vector Projection: The vector parallel to $\vec{a}$ with the magnitude of the $\vec{b}_a$ component.
$proj_{\ \vec{a}}\ \vec{b} = \frac{\vec{a} \cdot \vec{b}}{||\vec{a}||} * \frac{\vec{a}}{||\vec{a}||}$



Cross Product formula:

- $\vec{a} \times \vec{b} = ||\vec{a}||||\vec{b}|| \sin\theta \, \hat{n}$
- $\vec{a} \times \vec{b} = \begin{matrix} i & j & k \\ \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \vec{b}_x & \vec{b}_y & \vec{b}_z \end{matrix} = i \left[\vec{a}_y\vec{b}_z - \vec{a}_z\vec{b}_y\right] - j \left[\vec{a}_x\vec{b}_z - \vec{a}_z\vec{b}_x\right] + k \left[\vec{a}_x\vec{b}_y - \vec{a}_y\vec{b}_x\right]$

Properties of Cross Product:

- $if\ \vec{a} \times \vec{b} = 0,\ then\ \vec{a}\ is\ parallel\ to\ \vec{b}$
- $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$
- $\vec{a} \times (\vec{b} + \vec{c}) = (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c})$
- $k(\vec{a} \times \vec{b}) = \vec{b} \times (k\vec{a}) = \vec{a} \times (k\vec{b})$
- $\vec{a} \times \vec{0} = \vec{0} \times \vec{a} = \vec{0}$
- $\vec{a} \times \vec{a} = 0$

Area of a Parallelogram $= ||\vec{a} \times \vec{b}||$

Equation of a Line:

- $\vec{r} = (x_0 + at)\vec{i} + (y_0 + bt)\vec{j} + (z_0 + ct)\vec{k}$
- $\vec{r} = \langle x_0 + at, y_0 + bt, z_0 + ct \rangle$
- Parametric Form $\left\{x = x_0 + at \quad y = y_0 + bt \quad z = z_0 + ct\right\}$
- Symmetric Form $\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$

Intersection of Planes: Take the cross product of the normal vectors and then find the common point via substitution, plug vals into plane equation.

Equation of a Plane: $a(x - x_0) + b(y - y_z) + c(z - z_0) = 0,\ \hat{n} = \langle a, b, c \rangle$

Parallel, Skew, & Intersecting:

- $if\ \vec{a} \times \vec{b} = 0,\ parallel$
- Set equal & solve if a common point exists, intersecting
- Otherwise Skew

Limits of Vector Functions: Approach from x, y, y=x, etc... until the limit = 2 different values, therefore DNE.
Hint: Substitute denominator into 1 term & use l'hopital's.

Calculus of Vector Functions:

- $\frac{d}{dt}\{\vec{u} + \vec{v}\} = \vec{u}\,' + \vec{v}\,'$
- $\frac{d}{dt}\{k\vec{u}\} = k\vec{u}\,'$
- $\frac{d}{dt}\{f(t)\vec{u}\} = f(t)\vec{u}\,' + \vec{u}f'(t)$
- $\frac{d}{dt}\{\vec{u} \cdot \vec{v}\} = \vec{u} \cdot \vec{v}\,' + \vec{v} \cdot \vec{u}\,'$
- $\frac{d}{dt}\{\vec{u} \times \vec{v}\} = \vec{u} \times \vec{v}\,' + \vec{u}\,' \times \vec{v}$
- $\frac{d}{dt}\{\vec{u}(f(t))\} = \vec{u}\,'(f(t)) * f'(t)$

Arc Length: $L = \int_a^b \sqrt{f'(t)^2 + g'(t)^2 + h'(t)^2}dt$

Curvature:

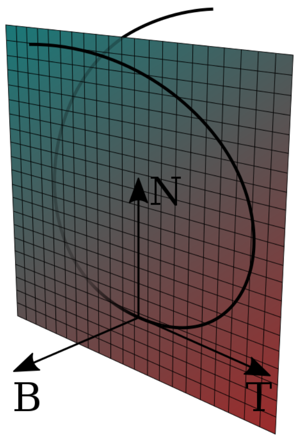
- $\kappa(t) = \frac{||\vec{T}\,'(t)||}{||\vec{r}\,'(t)||}$
- $\kappa(t) = \frac{||\vec{r}\,'(t) \times \vec{r}\,''(t)||}{||\vec{r}\,'(t)||^3}$
- $\kappa(x) = \frac{||f''(x)||}{||1+f'(x)^2||^{\frac{3}{2}}}$

TNB-Frames:

Unit Tangent Vector $\vec{T}(t) = \frac{\vec{r}\,'(t)}{||\vec{r}\,'(t)||}$

Unit Normal Vector $\vec{N}(t) = \frac{\vec{T}\,'(t)}{||\vec{T}\,'(t)||}$

Binormal Vector $\vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$



Osculating Plane: $\vec{B}_x(x - x_0) + \vec{B}_y(y - y_0) + \vec{B}_z(z - z_0) = 0$

Limits approaching via Axes: $\lim_{(x,y)\to(0,0)} \frac{f(x,y)}{g(x,y)}$
Approach from $x = 0$ & $y = 0$, then approach from the x *or* y ratio such that $g(x, y)$ becomes a single term, and both $f(x, y)$ & $g(x, y)$ use the same singular variable x *or* y. Now use l'hopital's, if any of the results conflict the limit does not exist. You are actively trying to prove the limit does not exist.

Partial Derivatives:

First Order Partials: $f_x = \frac{d}{dx}\{f(x, y)\}$, $f_y = \frac{d}{dy}\{f(x, y)\}$

Second Order Partials: $f_{xx} = \frac{d}{dx^2}\{f(x, y)\}$, $f_{yy} = \frac{d}{dy^2}\{f(x, y)\}$, $f_{xy} = \frac{d}{dy}\left\{\frac{d}{dx}\{f(x)\}\right\}$, $f_{yx} = \frac{d}{dx}\left\{\frac{d}{dy}\{f(x)\}\right\}$

Clairaut's Theorem: $if\ f_{xy}$ & f_{yx} *are continuous*, $f_{xy} = f_{yx}$

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– EXAM 2 NOTES

The following topics are covered:

- ☐ 4.4 - Tangent Planes & Linear Approximations
- ☐ 4.5 - Chain Rule
- ☐ 4.6 - Direction Derivatives & the Gradient Vector
- ☐ 4.7 - Maxima/Minima
- ☐ 5.1 - Double Integrals over Rectangular Regions
- ☐ 5.2 - Double Integrals over General Regions
- ☐ 5.3 - Double Integrals in Polar Coordinates
- ☐ 2.6 - Quadric Surfaces

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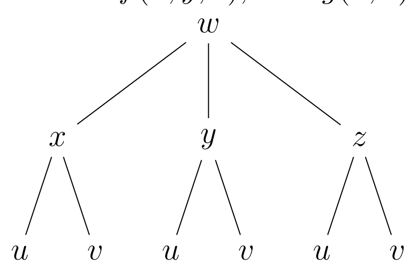
Tangent Plane: $L(x,y) = f(x_0,y_0) + f_x(x - x_0) + f_y(y - y_0)$

Linear Approximation: Input your point into the Tangent Plane formula to find the linear approximation.

Chain Rule: $\frac{dz}{dt} = \frac{dz}{dx} \cdot \frac{dx}{dt} + \frac{dz}{dy} \cdot \frac{dy}{dt}$

Don't forget the tree strategy for finding the derivative order.

ex. $w = f(x,y,z),\ x = g(u,v),\ y = h(u,v),\ z = j(u,v)$



Gradient Vector: $\nabla f(x,y) = \langle f_x(x,y), f_y(x,y) \rangle$

Directional Derivative:

- $D_{\vec{u}} f(x,y) = f_x(x,y)a + f_y(x,y)b,\ \vec{u} = \langle a, b \rangle$
- $D_{\vec{u}} f(x,y) = \langle f_x(x,y), f_y(x,y) \rangle \cdot \vec{u}$
- $D_{\vec{u}} f(x,y) = \nabla f(x,y) \cdot \vec{u}$

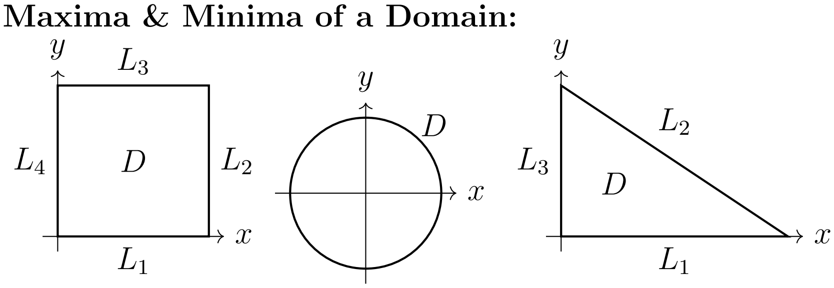
$\vec{u}$ is a unit vector that represents the direction that the directional derivative takes.

Finding the max & min:

The maximum value of the directional derivative represents the steepest path to the top of the surface. In the direct opposite direction lies the minimum value, which represents the steepest path downward.

$max = ||\nabla f(x,y)||\hat{u},\ min = -||\nabla f(x,y)||\hat{u}$

Maxima & Minima of a Domain:



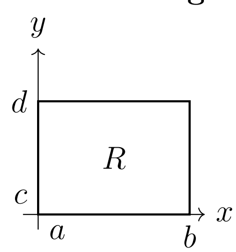
Take the partial derivatives $f_x(x,y)$ & $f_y(x,y)$, and set them equal to zero. Then substitute in the equation for each of the sides (ex.$L1, L2, L3$...), and once again set the partials equal to zero. This will find the critical points within the domain D.

Test each of these points along with the corners in the original equation to find the value of z, and with the below theorem to discover the type of extrema. Giving the local max/min, as well as absolute max/min, and saddle points.

$D = f_{xx}(x, y) \cdot f_{yy}(x, y) - [f_{xy}(x, y)]^2$

- $if\ D > 0\ \&\ f_{xx}(x_0, y_0) > 0,\ local\ min\ at\ (x_0, y_0)$
- $if\ D > 0\ \&\ f_{xx}(x_0, y_0) < 0,\ local\ max\ at\ (x_0, y_0)$
- $if\ D < 0,\ saddle\ point\ at\ (x_0, y_0)$
- $if\ D = 0,\ inconclusive$

Double Integrals over Rectangular Regions:

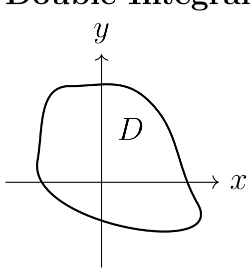


The double integral aka the volume below the surface $f(x, y)$ in the region R is equal to:

- $\iint_R f(x, y)dA = \int_a^b \left[\int_c^d f(x, y)dy\right] dx$
- $\iint_R f(x, y)dA = \int_c^d \left[\int_a^b f(x, y)dx\right] dy$
- In select cases: $\iint_R f(x, y)dA = \int f(x)dx \cdot \int f(y)dy$

Fubini's Theorem dictates that the order of integration does not matter. Tho one might be easier!

Double Integrals over General Regions:



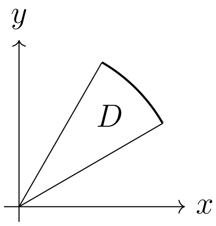
The double integral aka the volume below the surface $f(x, y)$ in the region D is equal to:

- $\iint_D f(x, y)dA = \int_a^b \left[\int_{g_1(x))}^{g_2(x)} f(x, y)dy\right] dx$
  $aka:\ left \to right,\ bottom \to top$

- $\iint_D f(x,y)dA = \int_c^d \left[\int_{h_1(y))}^{h_2(y)} f(x,y)dx\right] dy$
  $aka:\ bottom \to top,\ left \to right$

Fubini's Theorem dictates that the order of integration does not matter. Tho one might be easier!

Double Integrals in Polar Coordinates:

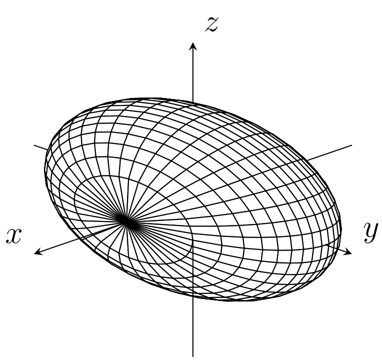


The double integral aka the volume below the surface $f(x,y)$ in the region D is equal to:

- $\iint_D f(rcos(\theta), rsin(\theta))dA = \int_\alpha^\beta \int_a^b f(rcos(\theta), rsin(\theta))rdrd\theta$

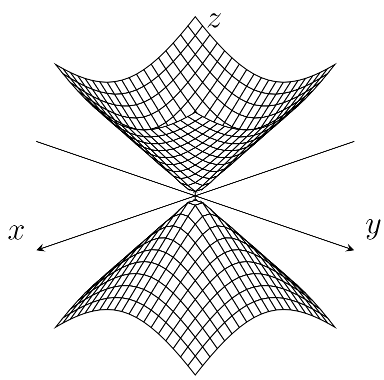
Quadric Surfaces:

Ellipsoid



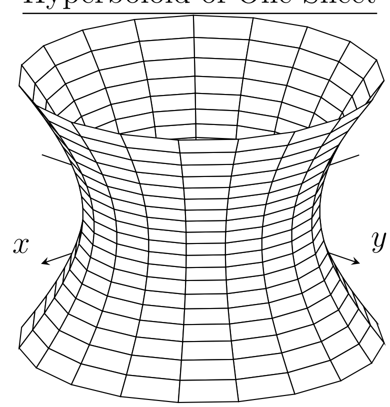
$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Cone



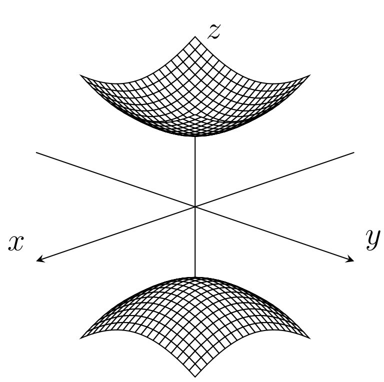
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$

Hyperboloid of One Sheet



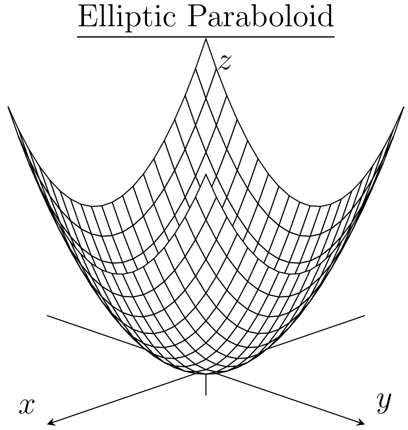
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2} + 1$

Hyperboloid of Two Sheets



$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2} - 1$

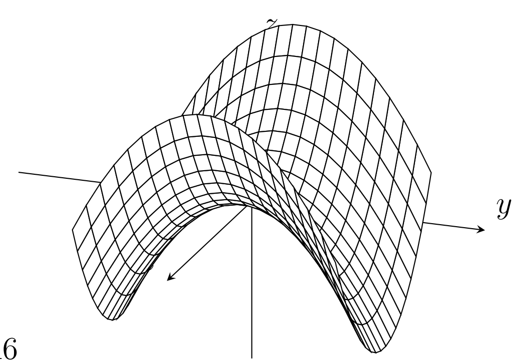
Elliptic Paraboloid



$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$

cell6

Hyperbolic Paraboloid



$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{z}{c}$

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– EXAM 3+ NOTES

The following topics are covered:

- ☐ 5.4 - Triple Integrals
- ☐ 2.7 - Cylindrical & Spherical Coordinates
- ☐ 5.5 - Triple Integrals in Cylindrical & Spherical Coordinates
- ☐ 6.1 - Vector Fields
- ☐ 6.2 - Line Integrals
- ☐ 6.3 - Conservative Vector Fields
- ☐ 6.4 - Green's Theorem
- ☐ 6.5 - Divergence & Curl
- ☐ 6.6 - Surface Integrals
- ☐ 6.7 - Stokes' Theorem
- ☐ 6.8 - Divergence Theorem

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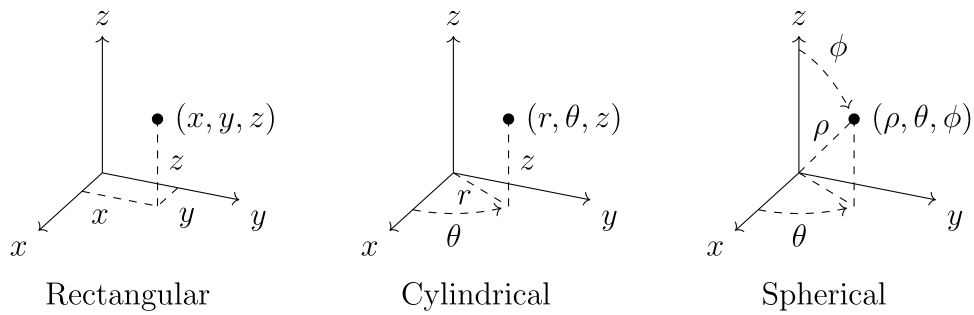
Coordinate Systems:



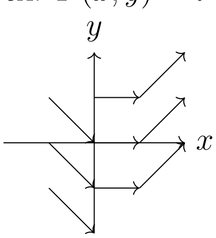
Triple Integrals:

- Rectangular Coordinates:
  $\iiint_\epsilon f(x,y,z)dV = \int_a^b \int_{g_1(x)}^{g_2(x)} \int_{h_1(x,y)}^{h_2(x,y)} f(x,y,z)dzdydx$
- Cylindrical Coordinates:
  $\iiint_\epsilon f(x,y,z)dV = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} \int_{g_1(rcos(\theta),rsin(\theta))}^{g_2(rcos(\theta),rsin(\theta))} f(rcos(\theta), rsin(\theta), z)dzrdrd\theta$
- Spherical Coordinates:
  $\iiint_\epsilon f(x,y,z)dV = \int_c^d \int_\alpha^\beta \int_a^b f(\rho sin(\phi)cos(\theta), \rho sin(\phi)sin(\theta), \rho cos(\phi))\rho^2 sin(\phi)d\rho d\theta d\phi$

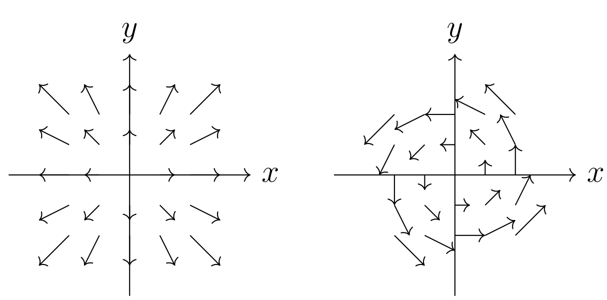
Vector Fields:

- $in\ \mathbb{R}^2\ \vec{F}(x,y) = P\vec{i} + Q\vec{j} = \langle P, Q\rangle$
- $in\ \mathbb{R}^3\ \vec{F}(x,y,z) = P\vec{i} + Q\vec{j} + R\vec{k} = \langle P, Q, R\rangle$

ex. $\vec{F}(x,y) = \vec{i} + x\vec{j} = \langle 1, x\rangle$:



Types of Vector Fields:



Radial ex. $\langle x, y\rangle$ Rotational ex. $\langle -y, x\rangle$

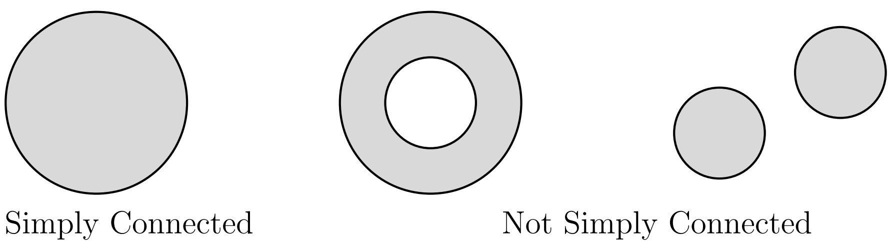
Line Integrals:

- With respect to x: $\int_C f(x,y)dx = \int_a^b f(x(t),y(t))x'(t)dt$
- With respect to y: $\int_C f(x,y)dy = \int_a^b f(x(t),y(t))y'(t)dt$
- Using potential(f): $\int_C F \cdot dr = \int_C \nabla f \cdot dr = f(b) - f(a)$

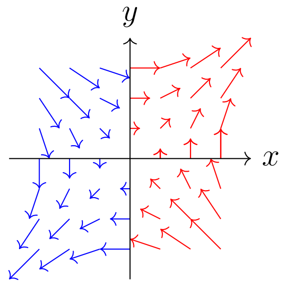
Finding the Potential Function (f) (vector field must be conservative):

1. If F is a Conservative Vector Field then $F = \nabla f$
2. $F = \langle f_x(x,y,z), f_y(x,y,z), f_z(x,y,z)\rangle$
3. Integrate: $\int f_x dx,\ \int f_y dy,\ \int f_z dz$
4. Determine sum that results in all first order partial derivatives
5. f = sum

Types of Regions:
Green's Theorem requires the region to be simply connected.



Conservative Vector Fields:
$\vec{F} = \nabla f$ *or* $\left[\frac{d\vec{P}}{dy} = \frac{d\vec{Q}}{dx}\ \&\ \frac{d\vec{Q}}{dz} = \frac{d\vec{R}}{dy}\ \&\ \frac{d\vec{R}}{dx} = \frac{d\vec{P}}{dz}\right]$ *or* $curl = 0$



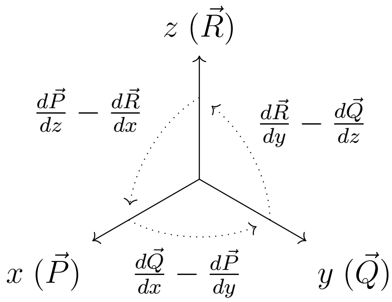
(notice how everything cancels)

Line Integrals in a Vector Field:
$\vec{F}(x,y) = P(x,y)\vec{i} + Q(x,y)\vec{j}\ in\ \mathbb{R}2$

Greens Theorem: The vector field must be over a simple connected region, and have continuous partials. With respect to x & y:

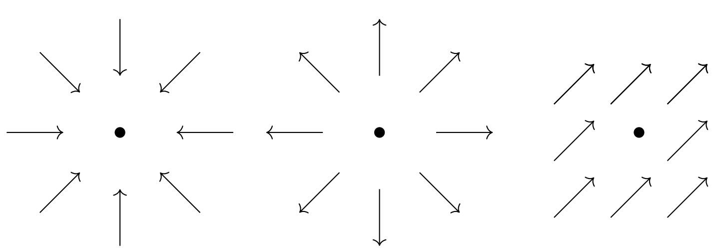
$$\int_C \vec{P}dx + \vec{Q}dy = \begin{cases} Flux = \iint_D (\vec{P}_x + \vec{Q}_y)dA,\ counterclockwise = negative(-) \\ Circulation = \iint_D (\vec{Q}_x - \vec{P}_y)dA,\ clockwise = negative(-) \end{cases}$$

Curl:
A measure of the "spin" or circulation of the vector field around a point.
$\left\langle \frac{d\vec{R}}{dy} - \frac{d\vec{Q}}{dz}, \frac{d\vec{P}}{dz} - \frac{d\vec{R}}{dx}, \frac{d\vec{Q}}{dx} - \frac{d\vec{P}}{dy} \right\rangle or \left\langle \frac{d\ \vec{\textbf{there}}}{d\ \textbf{here}} - \frac{d\ \vec{\textbf{here}}}{d\ \textbf{there}}, \ldots \right\rangle$
Curl of $\nabla = 0$, in conservative vector fields $\vec{F} = \nabla$, therefore curl of conservative vector fields is zero.



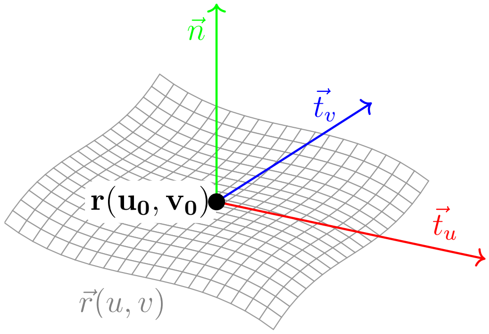
- $if\ curl = 0,\ vector\ field = irrotational\ \&\ conservative$
- $if\ curl \neq 0,\ vector\ field = rotational\ \&\ nonconservative$
- $if\ curl > 0,\ evaluated\ counterclockwise$
- $if\ curl < 0,\ evaluated\ clockwise$

Divergence:
$div\ F = \frac{d\vec{P}}{dx} + \frac{d\vec{Q}}{dy} + \frac{d\vec{R}}{dz}$
Source-Free vector fields are incompressible.



Sink $div < 0$ | Source $div > 0$ | Source-Free $div = 0$

Parameterizing a Surface:
$\vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v)\rangle$
A surface is smooth if $\vec{r}_u \times \vec{r}_v \neq \vec{0} \quad \forall u, v$. (fyi: $\forall = for\ all$)

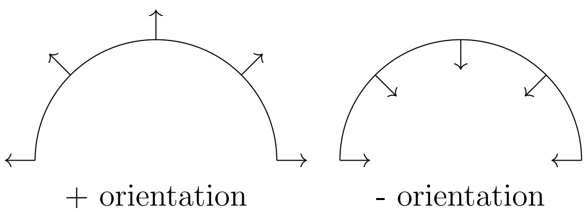


Surface Integral:
$\vec{r}(u,v) = surface\ parameterization,\ \vec{t} = tangent\ vector,\ \vec{n} = normal\ vector$

- $\iint_S f(x,y,z)dS = \iint_D f(\vec{r}(u,v)) \left\|\vec{t}_u \times \vec{t}_v\right\| dA$
- in a vector field: $\iint_S \vec{F} \cdot \vec{n}\ d\vec{s} = \iint_D \vec{F} \cdot (\vec{r}_u \times \vec{r}_v)dA$

Open vs. Closed Surfaces:
A Closed Surface completely encloses a three-dimensional region, an Open Surface does not.

Surface Orientation on Closed Surfaces:



Divergence Theorem:
For the surface integral of ϵ.
ϵ = a solid region in $\mathbb{R}3$.
S = a boundary surface with positive orientation.
$\vec{F}$ = vector field in $\mathbb{R}3$ & has continuous partials.

$\iint_S \vec{F} \cdot d\vec{s} = \iiint_\epsilon div\vec{F}\ dV$

Stokes' Theorem:
For a surface integral integral around curve C.
S = positive oriented smooth surface.
C = a boundary curve.
$\vec{F}$ = vector field in $\mathbb{R}3$ & has continuous partials.

$\int_C \vec{F} \cdot d\vec{r} = \iint_S curl\vec{F} \cdot \vec{n}\ ds = \iint_S cur\vec{F} \cdot (\vec{r}_u \times \vec{r}_v)\, dA$

Note: Stokes Theorem can be applied in both directions, but for testing purposes evaluate left to right.