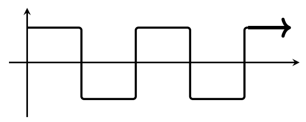
Fiber Optics

⚠ THIS IS NOT OFFICIAL, USE AT YOUR OWN RISK ⚠
ITEMS ARE NOT IN ORDER

Digital

Digital signals have discrete amplitude values aka levels. Signals are digitized via a sampling period and rounding to the nearest level.



Number of Levels = $2^{\text{number of bits}}$

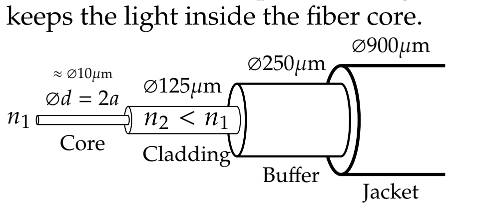
Bandwidth & Bit Rate

Modal Bandwidth (ΔM) ((Mb · km)/s, (MHz · km)/s) is a representation of the supported speed at which bits can travel through a fiber. It is the product of the bit rate B in megabits per second Mb/s and the length L in km. The bit rate can be considered as how many bits are transmitted per second, this can be converted to frequency (Hz) by considering bits unitless.

$$\Delta M = B \cdot L$$

Fiber Structure

Fibers have a core diameter d and core radius a. Numerical Aperture (NA) defines the cone of acceptance for light to enter the fiber. Total Internal Reflection keeps the light inside the fiber core.



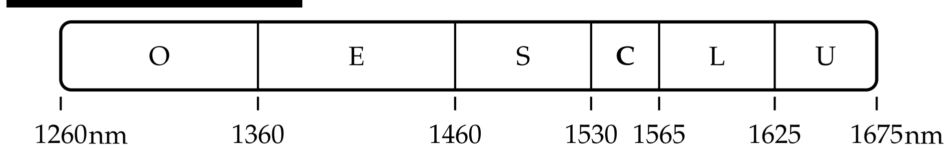
Stuff that makes fibers sad:
- Loss (attenuation α)
- Time Stretching (dispersion D)
- Frequency Stretching (nonlinearity)

Preform Drawing

When a preform is drawn/stretched into fiber, volume (V) stays constant.

$$V = L\pi\left(\frac{d}{2}\right)^2 \implies d^2_{\text{preform}} \cdot L_{\text{preform}} = d^2_{\text{fiber}} \cdot L_{\text{fiber}}, \quad \text{where} \begin{cases} d = \text{Outer Diameter} \\ L = \text{Length} \end{cases}$$

Communication Bands



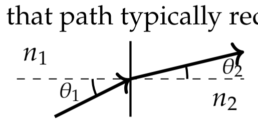
Index of Refraction & Snell's Law

When light encounters a medium it propagates as an electromagnetic oscillation of the medium itself. The effective "weight"of the medium creates a "phase kick-back"effectively slowing light down. Index of Refraction (IOR) (n) quantifies this effect.

$$n = \frac{c}{v} \geq 1 \qquad c = \text{speed of light} \qquad v = \text{propagation speed}$$

Snell's Law (refraction) describes how light takes the shortest temporal path through a medium. Due to changes in IOR that path typically requires an angular change.

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$



Total Internal Reflection (TIR)

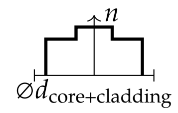
TIR occurs when rays encounter a high-to-low index boundary with an incident angle (θ) ≤ critical angle (θ_c). At the critical angle the light is refracted 90° and travels parallel to the incident surface. If the critical angle is exceeded the light will leave the medium.

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) \qquad \theta \leq \theta_c \therefore \text{Reflected}$$

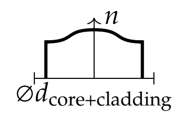
$$\theta > \theta_c \therefore \text{Transmitted \& Refracted} \qquad \theta = \theta_c \therefore \text{Travels Parallel to Interface}$$

Step v. Graded Index

For step index the core-cladding index difference (Δ) is 1 to 3% for multimode and 0.2 to 1% for single mode fibers. $\Delta = 1 - \frac{n_2}{n_1}$



Step-Index Delay $\sigma_s \simeq \frac{Ln_1\Delta}{2\sqrt{3}\cdot c}$



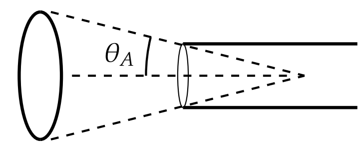
Graded-Index Delay $\sigma_s \simeq \frac{Ln_1\Delta^2}{20\sqrt{3}\cdot c}$

Numerical Aperature

Numerical Aperature (NA) is the light gathering ability of a optical element based on its acceptance/emitting cone of light.

$$\text{NA} = n_{\text{env}}\sin(\theta_A) = \sqrt{n_1^2 - n_2^2}$$

$\text{NA}_{\text{Lens}} < \text{NA}_{\text{Fiber}}$ $\quad d_{\text{Lens Spot}} \leq d_{\text{fiber}}$



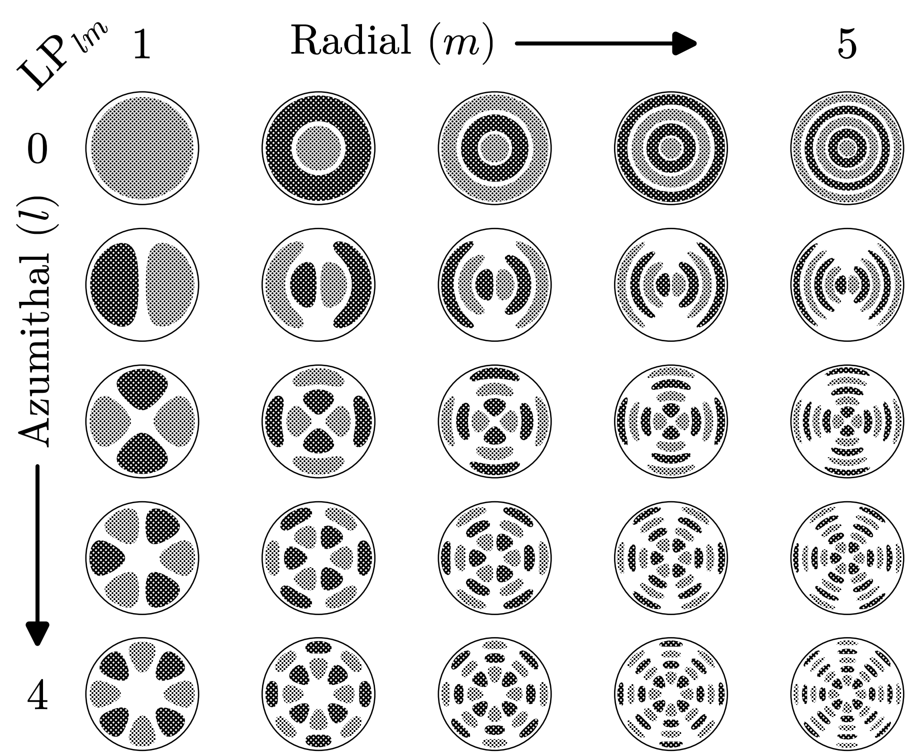
Modes

Number of allowed modes (M) (rounded down) is defined by a V number.

$$V = \pi\frac{d}{\lambda}\text{NA} \qquad M = \left\lfloor \frac{V^2}{2} \right\rfloor \qquad \begin{matrix} V < 2.405 \implies \text{Single Mode} \\ V \geq 2.405 \implies \text{Multi Mode} \end{matrix}$$

Mode Field Diameter (MFD) describes the distribution of power on the facet of a fiber. For a gaussian fiber of diameter d.

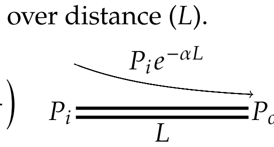
$$\text{MFD} = \frac{2\omega_0}{d} \quad \text{if } 1.2 \leq V \leq 2.4$$



Attenuation

Attenuation (α) describes the loss of received power (P_o) over distance (L).

$$\text{Power (dB)} = 10 \cdot \log_{10}\left(\frac{P_1}{P_2}\right) \qquad \alpha(\text{dB/km}) = \frac{10}{L}\log_{10}\left(\frac{P_i}{P_o}\right)$$



The output power (P_o) of a link can be calculated (in dB or dBm) by adding up P_i, G_{repeater}, & $L \cdot \alpha$. Pure silica has an attenuation of $\alpha(\lambda) = \alpha_0\left(\frac{\lambda_0}{\lambda}\right)^4$ With raighligh scattering being $\alpha_R = \frac{C}{\lambda^4}$ Note that dBm is the same scale as dB and simply denotes mW.

Group Velocity Dispersion (GVD)

Dispersion caused by a medium expressed as an ODE via Taylor expansion of propagation constant (β).

$$\text{Speed of Pulse } \beta_1 = \frac{1}{c}\left(n + \omega \cdot \frac{dn}{d\omega}\right) = \frac{1}{v_g}$$

$$\text{Broadening of Pulse } \beta_2 = \frac{1}{c}\left(2 \cdot \frac{dn}{d\omega} + \omega \cdot \frac{d^2n}{d\omega^2}\right)$$

$$\text{Dispersion Slope } \beta_3 = \frac{d\beta_2}{d\omega}$$

Waveguide Dispersion

Dispersion caused by differences in size and index between the core and the cladding.

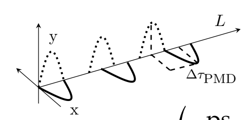
$$D_{wg} = -\frac{n_2\Delta}{c\lambda}\left[V\frac{d^2(Vb)}{dV^2}\right] \qquad b(V) = 1 - \frac{\left(1+\sqrt{2}\right)^2}{\left[1 + (4 + V^4)^{\frac{1}{4}}\right]^2}$$

if $D > 0 \therefore$ Blue Faster Than Red, if $D < 0 \therefore$ Red Faster Than Blue

Polarization Mode Dispersion (PMD)

Dispersion caused by difference in x & y polarization behavior.

$$\text{Time Delay } \Delta\tau_{\text{PMD}} = \left|\frac{L}{v_{gx}} - \frac{L}{v_{gy}}\right| = L\left|\beta_{1x} - \beta_{1y}\right| = L \cdot \delta\beta_1$$



Time Delay Variance/Pulse Period $\sigma_T = D_p \cdot \sqrt{L}$, where D_p is PMD parameter $\left(\frac{\text{ps}}{\sqrt{km}}\right)$

Dispersion Parameter D

Represents the combined dispersion of the medium & waveguide. However does not include polarization, intermodal, or intramodal dispersion.

$D > 0 \therefore$ Anomalous, $v_{\text{blue}} > v_{\text{red}}$ $\qquad D < 0 \therefore$ Normal, $v_{\text{red}} > v_{\text{blue}}$

$$D_{\text{total}} = D_{\text{medium}} + D_{\text{waveguide}} = \frac{1}{L[\text{km}]} \cdot \frac{\sigma[\text{ps}]}{\Delta\lambda[\text{nm}]} \implies \underbrace{\text{Pulse Width } \sigma[\text{ps}]}_{\text{Dispersed Pulse Duration}} = D(\lambda)L\Delta\lambda$$

$$\text{Time Bandwidth Product (TBP) (Gaussian)} = \Delta f \cdot \Delta t = 0.44$$

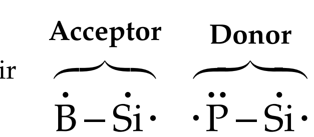
$$\text{Spectral Bandwidth (nm) } \Delta\lambda = \frac{c}{f^2}\Delta f = \frac{\lambda^2}{c}\Delta f$$

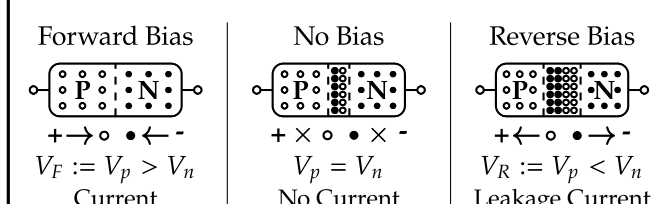
$\underbrace{\text{Pulse Duration (s) } \Delta t}_{\text{Input Pusle Width}}$ $\qquad$ Frequency Bandwidth (Hz) Δf

Semiconductor Fundamentals

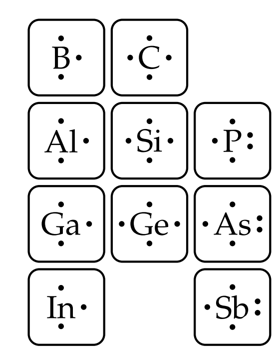
Semiconductors are materials who share properties of conductors & inductors, and can be "switched"between the two. As well they can be doped/biased to regionally contain more holes or more electrons.

The Octet Rule: atoms are most stable when their valence shells are filled with eight electrons.



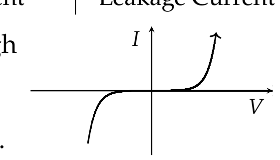


Common Elements



Diffusion: Passively from high to low concentration.

Drift: Driven by electric field.



Carriers & Doping

The output wavelength (λ) of a LED or laser is directly linked to the band-gap energy (E_g).

$$E = h\nu \equiv \frac{hc}{\lambda} \qquad n_i = K \cdot \exp\left(-\frac{E_g}{2 \cdot k_B \cdot T}\right) \qquad K = 2\left(\frac{2\pi k_B T}{h^2}\right)^{\frac{3}{2}} (m_e m_h)^{\frac{3}{4}}$$

By carefully selecting the ratio of acceptors, donors, and intrinsic materials the band-gap can be tuned via Vegard's Law.

$$E_{\text{InPAs}} = x \cdot E_{\text{InP}} + (1-x)E_{\text{InAs}} + b \cdot x(1-x)$$

b is the bowing parameter and describes the non-linear effect of the lattice.

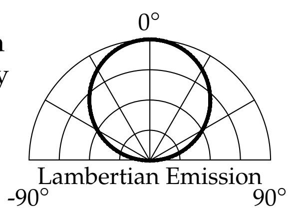
If a material has an equal number of holes and carriers it is intrinsic, otherwise it is extrinsic.

$$n_i^2 = n_0 \cdot p_0$$

Molecule	E_g (eV)	λ (nm)
InAs	0.3	4133
GaAs	1.4	886
$In_{0.53}Ga_{0.47}As$	0.75	1653
InP	1.3	954
GaP	2.3	539
$In_{.5}Ga_{.5}P$	1.9	652

Light Emitting Diodes (LEDs)

Holes and electrons recombine releasing light. When non-radiative recombination (R_n or τ_n) occurs energy is lost to heat.



$$L(\theta) = \frac{I_0 \cdot \cos(\theta)}{\cos(\theta)} = I_0 \qquad R_r = \frac{n}{\tau_r} \qquad R_n = \frac{n}{\tau_n}$$

$$\text{Total Recombination Time } \tau_{\text{bulk}} = \left(\frac{1}{\tau_{\text{SRH}}} + \frac{1}{\tau_{\text{Auger}}} + \frac{1}{\tau_{\text{radiative}}}\right)^{-1}$$

$$\text{Internal Quantum Efficiency (IEQ) } \eta_{int} = \frac{R_r}{R_r + R_n} \equiv \frac{1}{1 + \frac{\tau_r}{\tau_n}}$$

$$\text{Internal Optical Power } P_{int} = \eta_{int}\left(\frac{I}{q}\right)h\nu \qquad \phi_c = \sin^{-1}\left(\frac{n_{\text{clad}}}{n_{\text{core}}}\right)$$

$$\text{Light Extraction Efficiency (LEE)} = \frac{1}{4\pi}\int_0^{\phi_c} T(\phi) \cdot 2\pi \sin(\phi) d\phi$$

$$\text{External Quantum Efficiency EQE} = \text{LEE} \cdot \text{IQE}$$

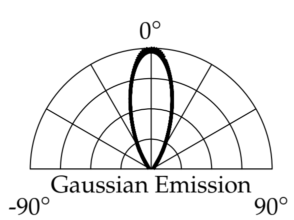
$$\text{Power Emitted} = P_{int} \cdot \text{LEE}$$

At normal incidence:

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 \qquad T = 1 - R \qquad \text{LEE} = \frac{1}{n_1 (n_1 + 1)^2}$$

Laser Diodes

Laser diodes emit highly directional coherent light when operated above the lasing threshold.



For a cavity at resonance:

$$\text{FSR}|_\gamma = \Delta\nu = \frac{c}{2n_{\text{eff}}L} \qquad \text{FSR}|_\lambda = \Delta\lambda \frac{\lambda^2}{2n_{\text{eff}}L}$$

$$\text{Lasing Condition } \Gamma \cdot g = \alpha_{\text{medium}} + \frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right)$$

$$\text{Lasing Threshold } g_{th} = \beta_{th} \cdot J_{th}$$

Overlap Factor: Γ Material Loss: α
Gain Factor: β Current Density: J
Cavity Reflectivity: R_1 & R_2 Cavity Length: L

Source to Fiber Coupling

$$\text{Radiance of Fiber } L_{\text{fiber}} = n_{\text{fiber}} \cdot A_{\text{fiber}} \cdot \sin(\theta_{\text{fiber}})$$

$$\text{Radiance of LED } L_{\text{LED}} = n_{\text{LED}} \cdot A_{\text{LED}} \cdot \sin(\theta_{\text{LED}})$$

$$\text{Efficient Coupling: } L_{\text{LED}} \leq L_{\text{fiber}} \qquad \eta_{\text{coupling}} = \frac{P_{\text{Fiber}}}{P_{\text{Source}}}$$

Modulating Sources

LEDs:

$$P(\omega) = P_o(1 + \omega \cdot \tau_i)^{-\frac{1}{2}}$$

Laser Diodes:

$$\text{Average Photon Lifetime } R_{stim} = \sigma_{21} \cdot \left(\frac{I_{sig}}{h \cdot \nu_{sig}}\right) \cdot n_2$$

$$\underbrace{\text{Average time photon stays in cavity } \tau_{ph}^{-1} = \frac{c}{n} g_{th}}_{\text{ultimate limit on modulation speed}}$$

Mach-Zehnder Modulator (MZM), device which uses electro-optic effects to modulate light via change of phase.

$$P_o = \frac{1}{2}P_i\left[1 + \cos(\Delta\phi)\right], \text{ where } \phi \text{ is phase.}$$

Electro-Optic Effect

Allows the modulation of material characteristics via a change in voltage. Pockel's Linear-Effect is where non-centrosymmetric crystals expand, contract, or change index according to applied voltage.

$$\Delta\phi = \frac{2\pi}{\lambda} \cdot n^3 r_{ij} \cdot L \cdot V$$

r_{ij} is the electro-optic tensor and describes the index change for different electric field orientations.

$$\text{LiNbO}_3 \begin{cases} r_{13} = 8.6\text{pm/V} \\ r_{22} = 3.4\text{pm/V} \\ r_{33} = 30.8\text{pm/V} \end{cases}, \quad r = \begin{bmatrix} 0 & 0 & 0 & 0 & r_{22} & 0 \\ 0 & 0 & 0 & r_{22} & 0 & 0 \\ r_{13} & r_{13} & r_{33} & 0 & 0 & 0 \end{bmatrix} \begin{matrix} E_x \\ E_y \\ E_z \end{matrix}$$

$$\quad x \quad y \quad z \quad yz \quad zx \quad xy$$

$$\text{Voltage for } \pi \text{ phase change } V_\pi = \frac{\lambda d}{n^3 r_{33} L\Gamma}$$

Kerr Nonlinear-Effect: Electric field causes a change in index of centrosymmetric media. The effect is weaker than Pockel's but occurs proportional to the square of the electric field rather than linear.

Noise & SNR

$$\text{Shot (Current) Noise } \langle i_{\text{shot}} \rangle = 2q i_p B_e \qquad \text{Thermal Noise } \langle i_{\text{th}} \rangle = \frac{4k_B T}{R_L} B_e$$

$$\text{Signal to Noise Ratio (SNR)} = \frac{i^2_{\text{signal}}}{i^2_{th} + i^2_{shot}}$$

$$\text{Noise Equivalent Power (NEP) (SNR:=1)} = \frac{P_{\min}}{\sqrt{B}}$$

$$\text{Bit Error Rate (BER)} = \frac{N_{\text{errors}}}{N_{\text{pulses}}} \qquad N_{\text{photons}} = \frac{P \cdot \tau}{h \cdot f}$$

$$\text{Probability of Pulse at Incident Power } P_r(n) = \overline{N}^n \frac{e^{-\overline{N}}}{n!} \qquad \overline{N} = \frac{\eta\lambda}{hc}E$$

$$\text{Power to Achieve BER at bitrate } P_{avg} = \frac{E_{min}B}{2}$$

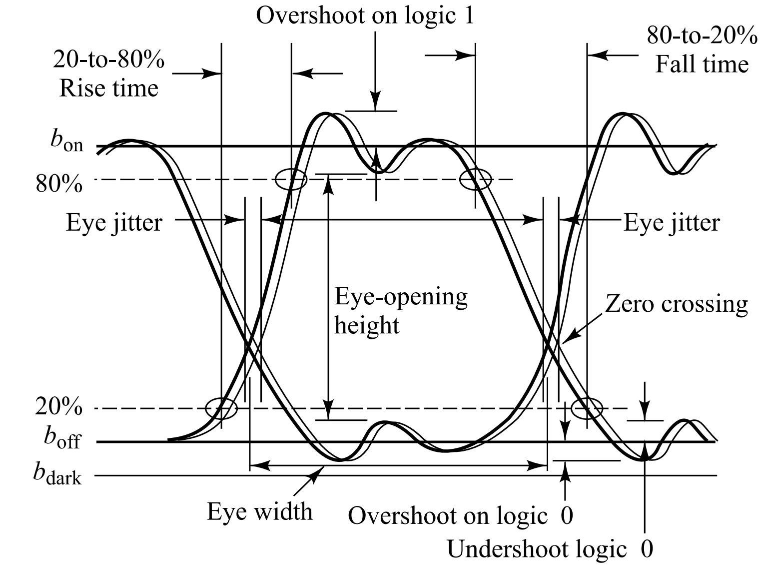
Photodetectors

$$\text{Responsivity } R = \frac{i_p}{P_{in}} = \frac{\eta q}{h\nu} \qquad \text{Response Speed } B_c = \frac{1}{2\pi R_T C_t}$$

	Si	**Ge**	**InGaAs**
Wavelengths (nm)	400 to 1100	800 to 1650	1100 to 1700
Responsivity (A/W)	0.4 - 0.6	0.4 - 0.5	0.75 - 0.95
Dark Current (nA)	1 - 10	50 - 500	0.5 - 2.0
Rise Time (ns)	0.5 - 1	0.1 - 0.5	0.05 - 0.5
Bandwidth (GHz)	0.3 - 0.7	0.5 - 3	1 - 2
Bias Voltage (V)	5	5 - 10	5

The Eye Diagram

Eye diagrams displaying the noise and dispersion qualities of a fiber optic system by overlaying thousands of individual waveforms.



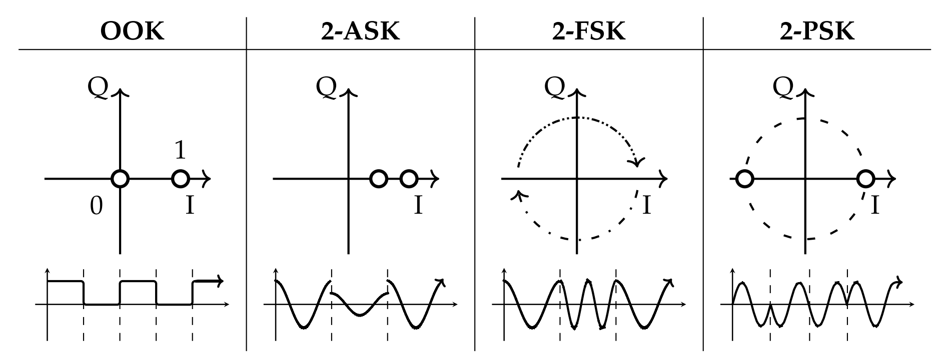
Keiser, G. (2021). Optical Receiver Operation. ISBN 978-981-33-4664-2

Modulation

Data signals contain an amplitude, frequency, and phase. Each of those components can be modulated forming the basic ASK/OOK, FSK, & PSK modulation formats.

IQ modulation is a method of representing any modulation format using complex numbers (I ($\mathbb{R}$) & Q ($i\mathbb{R}$). The graph of such is called a constellation chart.

$$s(t) = I(t) \cdot \cos(2\pi f_c t) - Q(t) \cdot \sin(2\pi f_c t)$$



Format names are vague and overlap, the waveform is best understood through the carrier and the constellation chart. Generally the more nodes the more data throughput but with higher loss.

Modulation Formats

Format	bits/symbol	Power Penalty (dB)	Use Case
ASK/OOK	1	0	Basic links
PAM-4	2	4.8	Datacenters
BPSK	1	4	Robust/satellite
QPSK	2	0	Long-haul
16-QAM	4	4-6	Metro links
64-QAM	6	7-8	Short links
256-QAM	8	12-13	High-capacity
1024-QAM	10	15	Short/high-capacity
4096-QAM	12	18	Real-time streaming

$$R_{\text{bits}} = R_{\text{symbol}} \cdot \frac{\text{bits}}{\text{symbol}} \quad \text{where } R_{\text{symbol}} \text{ is the baud-rate/symbols-per-second}$$

Multiplexing and Demultiplexing

Multiplexing is the process of combining multiple signal wavelengths within a single fiber via Wavelength Division Multiplexers (WDMs).

ITU channel recommendations are:

Course-WDM - 20nm Spacing
Dense-WDM - 12.5, 25, 50, 100 GHz Spacing

Number of Channels = $\frac{\text{Range}}{\text{Spacing}}$

Fiber Bragg Gratings (FBG) are used to selectively reflect and pass wavelengths via interference caused by a index based diffraction grating. They can be used to multiplex and demultiplex systems.

$\lambda_{\text{Bragg}} = 2n_{avg} \cdot \Lambda$ where Λ is the grating period

Splicing & Cleaving

Splicing (Power Loss)			Cleaving (Back Reflection)			
Fusion	V-Grooved	Elastic-tube	Flat	PC	UPC	APC
0.01 dB	0.1 - 0.2 dB	0.5 - 1 dB	<-30 dB	<-35 dB	<-55dB	<-65 dB
Melts	Ends Meet	Ends Aligned	Flat	Filleted	Rounded	Angled 8°

Useful Constants

$$h = 6.6262 \times 10^{-34}\text{J} \cdot \text{s} = 4.1357 \times 10^{-15}\text{eV} \cdot \text{s}$$

$$1\text{eV} = 1.602 \times 10^{-19}\text{J}$$

$$\text{Angstrom} = \text{Å} = 10^{-10}\text{m}$$

$$k_B = 8.617 \times 10^{-5}\text{eV/K} = 1.38 \times 10^{-23}\text{J/K}$$

$$c = 299,792,458\text{m/s}$$

SI

Base Units

Quantity	Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Thermodynamic Temperature	kelvin	K
Amount of substance	mole	mol
Luminous Intensity	candela	cd

Derived Units

Quantity	Unit (Symbol)	Formula
Frequency	hertz (Hz)	s^{-1}
Force	newton (N)	$kg \cdot m/s^2$
Energy or work	joule (J)	$N \cdot m$
Power	watt (W)	J/s
Electric Charge	coulomb (C)	$A \cdot s$
Electric potential	volt (V)	J/C
Electrical Resistance	ohm (Ω)	V/A
Electrical Conductance	siemens (S)	A/V
Electrical Capacitance	farad (F)	C/V
Magnetic Flux	weber (Wb)	$V \cdot s$
Inductance	henry (H)	Wb/A

Prefixes

Prefix		Power
zetta-	(Z)	10^{21}
exa-	(E)	10^{18}
peta-	(P)	10^{15}
tera-	(T)	10^{12}
giga-	(G)	10^{9}
mega-	(M)	10^{6}
kilo-	(k)	10^{3}
hecto-	(h)	10^{2}
deka-	(da)	10^{1}
– Base –		
deci-	(d)	10^{-1}
centi-	(c)	10^{-2}
milli-	(m)	10^{-3}
micro-	(μ)	10^{-6}
nano-	(n)	10^{-9}
pico-	(p)	10^{-12}
fento-	(f)	10^{-15}
atto-	(a)	10^{-18}
zepto-	(z)	10^{-21}

♡ Made by Kiva M. & Ollie M. ♡
♡ Check out more of my work at kivamccr.xyz ♡