Foundations of Photonics

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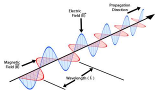
——— ITEMS ARE NOT IN ORDER ———

Light

Photons, a particle/wave duality that transfers energy via an electromagnetic field and does not require a medium to propagate (Unlike mechanical waves which do).

Consists of a electric field ($\vec{E}$) and magnetic field ($\vec{B}$) which are perpendicular to each other, a travel along the direction of propagation.



1D Waves & Propagation

Something oscillating with time, and in some cases moving in space, described as a function $\psi(x,t)$.

The general wave equation is: $\psi(x,t) = A \cdot \sin(k \cdot x - \omega \cdot t + \varepsilon)$.

The phase term is $\phi(x,t) = (k \cdot x - \omega \cdot t + \varepsilon) \implies \psi(x,t) = A\sin(\phi)$
A complete cycle / wavelength corresponds to $\phi = 2\pi$.

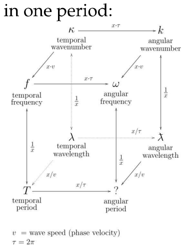
Waves must satisfy the scaler wave equation: $\frac{\partial^2\psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$.

Propagating: Forward $\psi(x,t) = f(x - v \cdot t)$ or Backward $\psi(x,t) = f(x + v \cdot t)$

Harmonic waves are waves which travel one wavelength in one period:
$\psi(x,t) = A\sin k(x - v \cdot t)$

Variables

Wavelength (λ) (m)	Temporal Frequency (f) (Hz)
Speed of Wave / Phase Velocity (v) (m/s)	Angular Frequency (ω) (rad/s)
Temporal Wavenumber (κ) (m^{-1})	Phase (ϕ) (rad)
Amplitude (A) (V/m or A/m)	Angular Wavenumber / Propagation Constant (k) (rad/m)
Temporal Period (T) (s)	Initial Phase (ε) (rad)



$$\vec{k} \times \vec{E} = \omega\vec{B}$$

The Speed of Light in Free Space: $c = 299,792,458\text{m/s} \approx \boxed{3 \times 10^8\text{m/s}}$

Validating Waves

Generally waves can be verified using the following equation:

$$\frac{\partial^2\psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \quad \text{or} \quad \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2\psi}{dt^2}$$

- Traveling waves must have a term in the phase of the form $\pm\omega \cdot t$.
- Waves must have a spatial component of the form $k \cdot \vec{u}$. $\vec{u}$ is a unit vector.
- Wave must not change in amplitude over distance or time to be considered valid in our case. Therefore the following is not valid:
  $\psi(x,t) = \frac{1}{z \cdot t}\cos(k \cdot z - \omega \cdot t)$

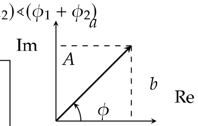
Complex Numbers

$$i = \sqrt{-1} \quad \& \quad \text{rad} \to {}^\circ = \text{Crad} \cdot \frac{180^\circ}{\pi\text{rad}} = C^\circ$$

Form	Equation	Conversion
Rectangular	$a + ib$	To polar: $A = \sqrt{a^2 + b^2}$, $\angle\phi = \tan^{-1}(\frac{b}{a})$
Polar	$Ae^{i\phi} = A\angle\phi$	To rectangular: $a = Acos(\phi)$, $b = Asin(\phi)$

Operation	Property
Addition	$(a_1 + ib_1) + (a_2 + ib_2) = (a_1 + a_2) + i(b_1 + b_2)$
Subtraction	$(a_1 + ib_1) - (a_2 + ib_2) = (a_1 - a_2) + i(b_1 - b_2)$
Multiplication	$A_1\angle\phi_1 \cdot A_2\angle\phi_2 = (A_1 \cdot A_2)\angle(\phi_1 + \phi_2)$
Division	$\frac{A_1\angle\phi_1}{A_2\angle\phi_2} = \frac{A_1}{A_2}\angle(\phi_1 - \phi_2)$

deg	0°	30°	45°	60°	90°
rad	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Und.



$$1 + i = \sqrt{2}\angle 45^\circ$$
$$\{a + ib\}^* = a - ib$$
$$\frac{1}{i} = -i$$

Euler's Formula: $e^{i\phi} = \cos\phi + i\sin\phi$, $\cos(\phi) = \frac{e^{i\phi} + e^{-i\phi}}{2}$, & $\sin(\phi) = \frac{e^{i\phi} - e^{-i\phi}}{2i}$

The wave equation: $\cos(kx \pm \omega t + \epsilon) \to \text{Re}\left[Ae^{\pm i(kx - \omega t + \epsilon)}\right]$

Propagating: Forward $\psi(x,t) = \text{Re}\left[Ae^{i(kx - \omega t + \epsilon)}\right] \to Ae^{i(kx + \epsilon)}$
or Backward $\psi(x,t) = \text{Re}\left[Ae^{-i(kx - \omega t + \epsilon)}\right] \to \underbrace{Ae^{-i(kx + \epsilon)}}_{\text{complex}}$

Warning time dependence can only be recovered if:

- Typically addition and/or subtraction.
- Multiplication and/or division are by a real quantity.
- Differentiation and/or integration are by a real variable.
- Avoid vector dot and cross product like the plague.
- Be aware of the pointing vector S = E × H.

To recover consider the following example:

$$\psi(x) = Ae^{i(kx+\varepsilon)} + Ae^{-i(kx+\varepsilon)} = 2A\cos(kx + \varepsilon)$$
$$\xrightarrow{\text{recover time dependance}} 2A\cos(kx + \varepsilon)\cos(\omega t)$$

Plane/Spherical Waves & Wavefronts

Plane waves are a series of equally spaced (constant phase) wavefronts, which are planes whose normal axis is the direction of propagation (perpendicular) and are located at the minima and maxima of a wave. These waves can propagate.

$$\vec{\psi}(\vec{r},t) = \hat{A}\sin(\vec{k} \cdot \vec{r} - \omega \cdot t) = \hat{A}e^{i(\vec{k} \cdot \vec{r} - \omega \cdot t)}$$

$$\vec{k} = k_x\hat{i} + k_y\hat{j} + k_z\hat{k}, \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}, \quad \& \quad \vec{k} \cdot \vec{r} = \text{constant}$$

$$|\vec{k}| = \frac{2\pi}{\lambda} = \sqrt{k_x^2 + k_y^2 + k_z^2} \quad \& \quad \alpha^2 + \beta^2 + \gamma^2 = 1$$

Cartesian form: $\vec{\psi}(x,y,z,t) = \hat{A}e^{k_x x + k_y y + k_z z - \omega \cdot t} = \hat{A}e^{k(\alpha x + \beta y + \gamma z) - \omega \cdot t)}$

Spherical waves are similar to plane waves however they radiate outward from a source, or example ripples from a falling object.

$$\psi(x,y,z,t) = A\frac{\cos(k \cdot r - \omega \cdot t}{r}$$

3D Waves

A wave who propagates in a direction in 3D space (x,y,z):

$$\psi(x,y,z,t) = A\cos(k_x x + k_y y + k_z z - \omega t) = A\cos\left(k\left(\frac{\alpha x + \beta y + \gamma z}{\sqrt{\alpha^2 + \beta^2 + \gamma^2}}\right) - \omega t\right)$$

$k = \frac{2\pi}{\lambda} = \sqrt{k_x^2 + k_y^2 + k_z^2}$ & $\frac{\alpha x + \beta y + \gamma z}{\sqrt{\alpha^2 + \beta^2 + \gamma^2}}$ is the unit vector of our direction of travel.

3D waves must satisfy the 3D scaler wave equation: $\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2\psi}{dt^2}$

The Laplacian operator is $\nabla^2\psi$ where: $\nabla^2 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial}{\partial z^2}$

To verify whether two waves are parallel or perpendicular:

- If the cross product = 0 then the two vectors are parallel:
  $$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \vec{b}_x & \vec{b}_y & \vec{b}_z \end{vmatrix} = \hat{i}\left[\vec{a}_y\vec{b}_z - \vec{a}_z\vec{b}_y\right] - \hat{j}\left[\vec{a}_x\vec{b}_z - \vec{a}_z\vec{b}_x\right] + \hat{k}\left[\vec{a}_x\vec{b}_y - \vec{a}_y\vec{b}_x\right]$$
- If the dot product = 0 then the two vectors are perpendicular:
  $\vec{a} \cdot \vec{b} = \vec{a}_x\vec{b}_x + \vec{a}_y\vec{b}_y + \vec{a}_z\vec{b}_z$

Superposition

The combination of two or more waves.

- When two waves are in the same region of space / overlapping $\psi_{\text{total}} = \psi_1 + \psi_2$.
- When they don't overlap they pass through unaffected.
- The disturbance only occurs on the overlap.

Interference: Constructive $\psi_1 + \psi_2 > \psi_1$ and ψ_2 or Destructive $\psi_1 + \psi_2 = 0$

Power: $P = E^2$ and In a transparent dielectric medium: $v = c/n$

To solve $\psi_1 + \psi_2 + \cdots + \psi_n$, you can either compute using complex addition and subtraction or use the following trig. formulas.

- $\cos(A) + \cos(B) = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$
- $\sin(A) + \sin(B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$
- $\cos(A) - \cos(B) = -2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$
- $\sin(A) - \sin(B) = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

SI

Base Units

Quantity	Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Thermodynamic Temperature	kelvin	K
Amount of substance	mole	mol
Luminous Intensity	candela	cd

Derived Units

Quantity	Unit (Symbol)	Formula
Frequency	hertz (Hz)	s^{-1}
Force	newton (N)	$kg \cdot m/s^2$
Energy or work	joule (J)	$N \cdot m$
Power	watt (W)	J/s
Electric Charge	coulomb (C)	$A \cdot s$
Electric potential	volt (V)	J/C
Electrical Resistance	ohm (Ω)	V/A
Electrical Conductance	siemens (S)	A/V
Electrical Capacitance	farad (F)	C/V
Magnetic Flux	weber (Wb)	$V \cdot s$
Inductance	henry (H)	Wb/A

Prefixes

Prefix		Power
zetta-	(Z)	10^{21}
exa-	(E)	10^{18}
peta-	(P)	10^{15}
tera-	(T)	10^{12}
giga-	(G)	10^{9}
mega-	(M)	10^{6}
kilo-	(k)	10^{3}
hecto-	(h)	10^{2}
deka-	(da)	10^{1}
– Base –		
deci-	(d)	10^{-1}
centi-	(c)	10^{-2}
milli-	(m)	10^{-3}
micro-	(μ)	10^{-6}
nano-	(n)	10^{-9}
pico-	(p)	10^{-12}
fento-	(f)	10^{-15}
atto-	(a)	10^{-18}
zepto-	(z)	10^{-21}

♡ Made by Kiva M. and others??? (pretty please) ♡