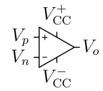
Linear Circuits II [Universal Method]

THIS IS NOT OFFICIAL, USE AT YOUR OWN RISK
ITEMS ARE NOT IN ORDER

Op-Amps

An Operational Amplifier (Op-Amp) changes the voltage, current, and/or power of a signal. Operations include:

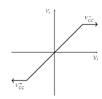
Sum	Differentiation $\frac{\partial}{\partial t}$	Scaling
Difference	Integration $\int \partial t$	Sign Changes



An Ideal Op-Amp makes the following assumptions:

$V_p = V_n$ (Virtual Short) $\quad i_p = i_n := 0 \quad i_o = -(i_{CC+} + i_{CC-}) \neq 0 \quad R_{\text{In}} = R_{eq}$ (internal)

$$\text{Gain} = A = \frac{V_o}{V_{\text{In}}} \quad V_o = \begin{cases} V_{CC}^{-} & A\left(V_p - V_n\right) < V_{CC}^{-} \\ A\left(V_p - V_n\right) & V_{CC}^{-} \leq A\left(V_p - V_n\right) \leq V_{CC}^{+} \\ V_{CC}^{+} & A\left(V_p - V_n\right) > V_{CC}^{+} \end{cases}$$

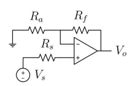


Solving Op-Amp Problems:

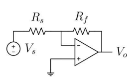
KCL at terminals V_p & V_n ($i_p = i_n := 0$) $\longrightarrow$ Set V_p equal to V_n $\longrightarrow$ Solve for V_o

Types of Op-Amps

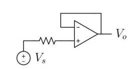
Non-Inverting: Amplifies the input voltage while preserving polarity.
Gain $= 1 + \frac{R_f}{R_a}$



Inverting: Amplifies the input voltage while reversing polarity.
Gain $= -\frac{R_f}{R_s}$

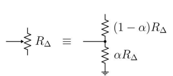


Buffer: Matches the impedance of the input to that of the output circuit.
Gain = 1

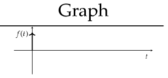
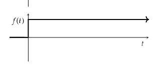
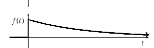
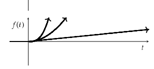
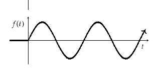
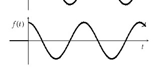
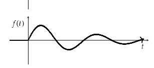
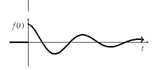


Potentiometers

A potentiometer is a variable resistor. The resistance can be described by an active resistor and a grounded/inactive resistor, whose ratio is defined by α.



Laplace Transforms

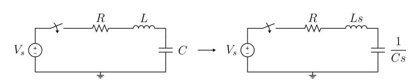
Type	$f(t)(t<0-)$	$F(s) = \mathscr{L}f(t)$	Graph
Impulse	$\delta(t)$	1	
Step	$u(t)$	$\frac{1}{s}, s>0$	
Exponential	e^{-at}	$\frac{1}{s+a}, s>a$	
Ramp	t^n	$\frac{n!}{s^{n+1}}, s>0$	
Sine	$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}, s>0$	
Cosine	$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}, s>0$	
Damped Ramp	$e^{-at}t^n$	$\frac{n!}{(s-a)^{n+1}}, s>a$	
Damped Sine	$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}, s>a$	
Damped Cosine	$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}, s>a$	

Properties of Laplace Transforms

Property	Transform Pair/Property
Linearity	$ax(t) + bv(t) \leftrightarrow aX(s) + bV(s)$
Right shift in time	$x(t-c)u(t-c) \leftrightarrow e^{-cs}X(s),\ c>0$
Time scaling	$x(at) \leftrightarrow \frac{1}{a}X\left(\frac{s}{a}\right),\ a>0$
Multiplication by a power of t	$t^N x(t) \leftrightarrow (-1)^N \frac{d^N}{ds^N}X(s),\ N = 1,2,\ldots$
Multiplication by an exponential	$e^{at}x(t) \leftrightarrow X(s-a),\ a$ real or complex
Multiplication by $\sin(\omega t)$	$x(t)\sin\omega t \leftrightarrow \frac{j}{2}\left[X(s+j\omega) - X(s-j\omega)\right]$
Multiplication by $cos(\omega t)$	$x(t)\sin\omega t \leftrightarrow \frac{j}{2}\left[X(s+j\omega) + X(s-j\omega)\right]$
Differentiation in the time domain	$\dot{x}(t) \leftrightarrow sX(s) - x(0)$
Second derivative	$\ddot{x} \leftrightarrow s^2X(s) - sx(0) - \dot{x}(0)$
N^{th} Derivative	$x^{(N)}(t) \leftrightarrow s^N X(s) - s^{N-1}x(0) - s^{N-2}\dot{x}(0)\ldots$ $-\ldots - sx^{(N-2)}(0) - x^{(N-1)}(0)$
Integration	$\int_0^t x(\lambda)d\lambda \leftrightarrow \frac{1}{s}X(s)$
Convolution	$x(t) * v(t) \leftrightarrow X(s)V(s)$
Initial-Value Theorem	$x(0) = \lim_{s\to\infty} sX(s)$; $\dot{x}(0) = \lim_{s\to\infty}\left[s^2X(s) - sx(0)\right]$; $x^{(N)}(0) = \lim_{s\to\infty}[s^{N+1}X(s) - s^N x(0)\ldots$ $-s^{N-1}\dot{x}(0) - \ldots - sx^{(N-1)}(0)]$
Final-Value Theorem	If $\lim_{t\to\infty} x(t)$ exists, then $\lim_{t\to\infty} x(t)\ldots$ $= \lim_{s\to 0} sX(s)$

Laplace Transform of RLC Circuits

Expressing circuits using Laplace allows the easy analysis of step/natural response in the RLC circuit, as we can use our standard techniques of analysis such as nodal, mesh, KCL, KVL, Ohm's Law, etc... in the s-domain. Effectively $s \equiv j\omega$. Remember if non-zero initial conditions exist to express them via LC source transformation.

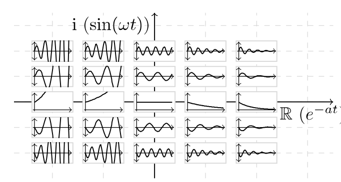


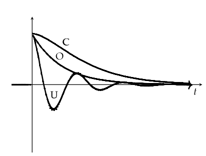
t-Domain Circuit $\xrightarrow{\mathscr{L}}$ s-Domain Calculation $\xrightarrow{\mathscr{L}^{-1}}$ t-Domain Answer

Transfer Functions & Damping

The transfer function $H(s)$ allows a generalized relationship for the input and output over any given range of values.

$$\underbrace{Y(s)}_{\text{Output}} = \underbrace{H(s)}_{\text{Function}} * \underbrace{X(s)}_{\text{Input}} \qquad \frac{V_{\text{Out}}}{V_{\text{In}}} = H(s) = \text{Gain}$$



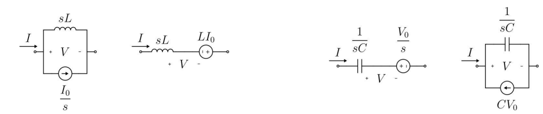


$\frac{(s-r_1)(s-r_2)}{(s-r_3)(s-r_4)}$ Poles: Where an equation is undefined ($y \to \infty$) ex. r_3 & r_4
Zeros: Where an equation is zero ($y = 0$) ex. r_1 & r_2

Over-Damped (O)	Critically-Damped (C)	Under-Damped (U)
$C_1e^{r_1t} + C_2e^{r_2t}$	$C_1e^{rt} + C_2te^{rt}$	$C_1e^{r_1t}\cos(\omega t) + C_2e^{r_2t}\sin(\omega t)$
Real Roots	Real Repeated Roots	Complex Roots

LC Transformation

Inductors & capacitors can undergo source transformation, just like resistors. However unlike resistors they can be sources themselves, in that case the voltage (capacitor) or current (inductor) source is their initial value V_0 or I_0.

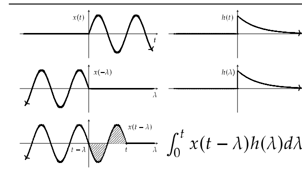


Convolution

A method to calculate the output signal ($y(t) \equiv Y(s)$) of a circuit from the circuit's transfer function ($h(t) \equiv H(s)$), input signal ($x(t) \equiv X(s)$), & signal duration (t).

Time Domain (t) $\quad y(t) = x(t) * h(t)$

Continuous

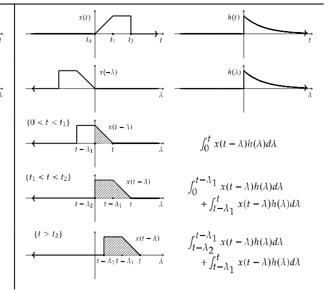


Laplace Domain (s)

$y(t) = x(t) * h(t) \Leftrightarrow Y(s) = X(s) \cdot H(s)$

$\mathscr{L}^{-1}\{Y(s)\} = y(t)$, don't evaluate

Piecewise



Draw your graphs. Identify the domain. Choose and flip a function. Shift function by t in λ space. Construct the integral(s). Solve the integral. If necessary plug in t.

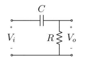
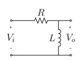
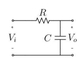
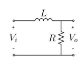
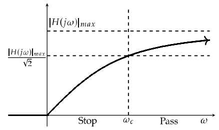
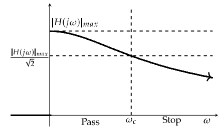
Sifting Property of the Impulse Function

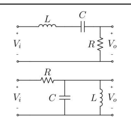
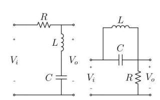
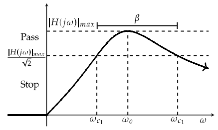
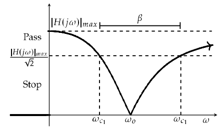
The Dirac/delta/impulse function is defined by: $\int_{-\infty}^{\infty}\delta(t)dt = 1$, this states that there is a vertical line with height infinity whose encompassing area equals one. The sift property creates a infinite vertical line at the point $(\lambda, 0)$ whose area is defined by the function $\phi(\lambda)$. This allows the extraction of a point value.

$$\int_{-\infty}^{\infty}\phi(t)\delta(t-\lambda)dt = \phi(\lambda)$$

Passive Filters

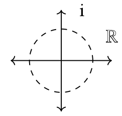
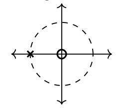
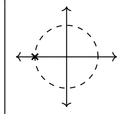
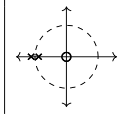
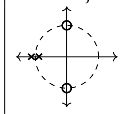
Passive filters are RLC devices that modulate the frequency of a system without adding power. Cutoff frequency (ω_c), where the output begins being considered zero. Bandwidth (β), width of domain in which the output is considered. Quality factor (Q), describes how purely resonant the system is.
$*$ Note all formulas below are in rad/s, Hz = $\frac{\text{rad/s}}{2\pi}$ & rad/s = $2\pi \cdot$ Hz.

	High Pass RC	High Pass RL	Low Pass RC	Low Pass RL
Circuit				
$H(j\omega)$	$\frac{j\omega}{j\omega+\omega_c}$		$\frac{\omega_c}{j\omega+\omega_c}$	
$\lvert H(j\omega)\rvert$	$\frac{1}{\sqrt{1+\left(\frac{\omega_c}{\omega}\right)^2}}$		$\frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}}$	
$\sphericalangle H(j\omega)$	$\tan^{-1}\left(\frac{\omega_c}{\omega}\right)$		$-\tan^{-1}\left(\frac{\omega}{\omega_c}\right)$	
ω_c	$\frac{1}{RC}$	$\frac{R}{L}$	$\frac{1}{RC}$	$\frac{R}{L}$
Graph				

	Band Pass	Band Reject
Circuit		
$H(j\omega)$	$\frac{\beta j\omega}{(j\omega)^2+\beta j\omega+\omega_o^2}$	$\frac{(j\omega)^2+\omega_o^2}{(j\omega)^2+\beta j\omega+\omega_o^2}$
$\lvert H(j\omega)\rvert$	$\frac{\omega\beta}{\sqrt{(\omega_o^2-\omega^2)^2+(\omega\beta)^2}}$	$\frac{-\omega^2+\omega_o^2}{\sqrt{(-\omega^2+\omega_o^2)^2+(\omega\beta)^2}}$
$\sphericalangle H(j\omega)$	$\frac{\pi}{2}-\tan^{-1}\left(\frac{\omega\beta}{\omega_o^2-\omega^2}\right)$	$-\tan^{-1}\left(\frac{\omega\beta}{\omega_o^2-\omega^2}\right)$
ω_o	$\sqrt{\omega_{c_1}\omega_{c_2}}=\sqrt{\frac{1}{LC}}$	
β	$\omega_{c_2}-\omega_{c_1}$ series: $\frac{R}{L}$, parallel: $\frac{1}{RC}$	
ω_{c_1,c_2}	$\pm\frac{\beta}{2}+\sqrt{\left(\frac{\beta}{2}\right)^2+\omega_o^2}$	
Q	$\frac{\omega_o}{\beta}$ series: $\sqrt{\frac{L}{CR^2}}$, parallel: $\frac{RC}{\sqrt{LC}}$	
Graph		

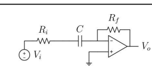
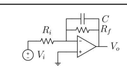
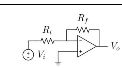
Pole Zero Plots

Filters both active and passive can be identified using the distribution and number of zeros (numerator) and poles (denominator). Displayed below are the simplest examples:

	High Pass	Low Pass	Band Pass	Band Reject
				
○ Zeros × Poles	Zeros: 1 Poles: 1 Order: 1	Zeros: 0 Poles: 1 Order: 1	Zeros: 1 Poles: 2 Order: 2	Zeros: 2 Poles: 2 Order: 2

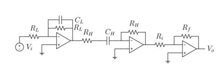
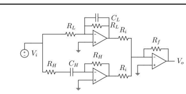
Active Filters

Active filters are RLC devices that modulate the frequency and power of a system. Whereas passive filters have a gain of $0 \leq A \leq 1$, active filters require power allowing gains of $K \geq 1$.

	High Pass (RC)	Low Pass (RC)	Gain
Circuit			
$H(j\omega)$	$-K\frac{j\omega}{j\omega+\omega_c}$	$-K\frac{\omega_c}{j\omega+\omega_c}$	$-K$
$\lvert H(j\omega)\rvert$	$K\frac{1}{\sqrt{1+\left(\frac{\omega_c}{\omega}\right)^2}}$	$K\frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}}$	K
$\sphericalangle H(j\omega)$	$\tan^{-1}\left(\frac{\omega_c}{\omega}\right)$	$-\tan^{-1}\left(\frac{\omega}{\omega_c}\right)$	0 rad
ω_c	$\frac{1}{R_iC}$	$\frac{1}{R_fC}$	N/A
K	$\frac{R_f}{R_i}$		

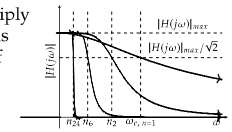
Cascade Filters

Cascade filters, are multiple filters linked together in series and/or parallel. As a result their effects stack, allowing the creation of band pass, band reject, and other complex filters from simpler filters such as high and low pass.

	Band Pass (RC)	Band Reject (RC)
Circuit		
$H(j\omega)$	$-K\frac{\beta j\omega}{(j\omega)^2+\beta j\omega+\omega_o^2}$	$K\frac{(j\omega)^2+\omega_o^2}{(j\omega)^2+\beta j\omega+\omega_o^2}$
$\lvert H(j\omega)\rvert$	$K\frac{\omega\beta}{\sqrt{(\omega_o^2-\omega^2)^2+(\omega\beta)^2}}$	$K\frac{-\omega^2+\omega_o^2}{\sqrt{(-\omega^2+\omega_o^2)^2+(\omega\beta)^2}}$
$\sphericalangle H(j\omega)$	$\frac{\pi}{2}-\tan^{-1}\left(\frac{\omega\beta}{\omega_o^2-\omega^2}\right)$	$-\tan^{-1}\left(\frac{\omega\beta}{\omega_o^2-\omega^2}\right)$
ω_o	$\sqrt{\omega_{c_1}\omega_{c_2}}=\frac{1}{\sqrt{R_LR_HC_LC_H}}$	
β	$\omega_{c_2}-\omega_{c_1}$	
ω_{c_1,c_2}	$\omega_{c_1}=\frac{1}{R_HC_H}$ $\omega_{c_2}=\frac{1}{R_LC_L}$	$\omega_{c_1}=\frac{1}{R_LC_L}$ $\omega_{c_2}=\frac{1}{R_HC_H}$
Q	$\frac{\omega_o}{\beta}=\frac{1}{(\omega_{c_2}-\omega_{c_1})\sqrt{R_LR_HC_LC_H}}$	
K	$\frac{R_f}{R_i}$	

Higher Order Filters

As filters are placed in cascade their effects multiply raising the order of the final transfer function. As order increases the filter will increase the roll-off slope (dB/decade) approaching an exclusively pass-band domain. Slope = $20n$dB/decade. Amplitude cutoff is at -3dB.



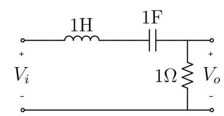
ex. n^{th}-order Low Pass Filter

$H(j\omega)=\left(\frac{\omega_c}{j\omega+\omega_c}\right)^n$ & $K_f=\sqrt{\sqrt[n]{2}-1}$

$V_{\text{dB}} = 20\log_{10}|H(j\omega)|$ $\quad P_{\text{dB}} = 10\log_{10}|H(j\omega)|$

Filter Scaling

Filter Scaling allows the creation of filters (both active & passive) using their prototypes in the form $R = 1\Omega$, $L = 1$H, and $C = 1$F ... and subsequently scaling those values to the desired ones, using the scalars K_m & K_f.

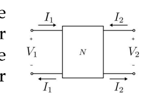


ex. Prototype Band-Pass

	Mag	Freq	Mag & Freq
Resistance (R')	K_mR	R	K_mR
Inductance (L')	K_mL	$\frac{1}{K_f}L$	$\frac{K_m}{K_f}L$
Capacitance (C')	$\frac{1}{K_m}C$	$\frac{1}{K_f}C$	$\frac{1}{K_mK_f}C$
Frequency (ω')	ω	$K_f\omega$	$K_f\omega$

Two Port Networks

Two Port Networks and their parameters allow us to relate their voltages to currents and make predictions about their behavior. The networks are typically characterized in the s-domain, and must follow a set of assumptions about their inner workings:



- No energy initially stored.
- No independent sources.
- Current in a port = Current out that port.
- All external connections must be made to <u>either</u> the input or output port.

Impedance

$$V_1 = z_{11}I_1 + z_{12}I_2 \qquad V_2 = z_{21}I_1 + z_{22}I_2 \qquad \begin{bmatrix}V_1\\V_2\end{bmatrix}=\begin{bmatrix}z_{11} & z_{12}\\ z_{21} & z_{22}\end{bmatrix}\begin{bmatrix}I_1\\I_2\end{bmatrix}$$

Admittance

$$I_1 = y_{11}V_1 + y_{12}V_2 \qquad I_2 = y_{21}V_1 + y_{22}V_2 \qquad \begin{bmatrix}I_1\\I_2\end{bmatrix}=\begin{bmatrix}y_{11} & y_{12}\\ y_{21} & y_{22}\end{bmatrix}\begin{bmatrix}V_1\\V_2\end{bmatrix}$$

Hybrid

$$V_1 = h_{11}I_1 + h_{12}V_2 \qquad I_2 = h_{21}I_1 + h_{22}V_2 \qquad \begin{bmatrix}V_1\\I_2\end{bmatrix}=\begin{bmatrix}h_{11} & h_{12}\\ h_{21} & h_{22}\end{bmatrix}\begin{bmatrix}I_1\\V_2\end{bmatrix}$$

Inverse Hybrid

$$I_1 = g_{11}V_1 + g_{12}I_2 \qquad V_2 = g_{21}V_1 + g_{22}I_2 \qquad \begin{bmatrix}I_1\\V_2\end{bmatrix}=\begin{bmatrix}g_{11} & g_{12}\\ g_{21} & g_{22}\end{bmatrix}\begin{bmatrix}V_1\\I_2\end{bmatrix}$$

Transmission

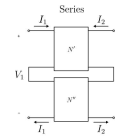
$$V_1 = a_{11}V_2 + a_{12}(-I_2) \qquad I_1 = a_{21}V_2 + a_{22}(-I_2) \qquad \begin{bmatrix}V_1\\I_1\end{bmatrix}=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}V_2\\-I_2\end{bmatrix}$$

Inverse Transmission

$$V_2 = b_{11}V_1 + b_{12}(-I_1) \qquad I_2 = b_{21}V_1 + b_{22}(-I_1) \qquad \begin{bmatrix}V_2\\I_2\end{bmatrix}=\begin{bmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\end{bmatrix}\begin{bmatrix}V_1\\-I_1\end{bmatrix}$$

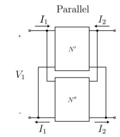
Interconnection of Networks

Series



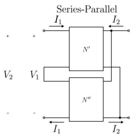
$[z]=[z]'+[z]''$

Parallel



$[y]=[y]'+[y]''$

Series-Parallel

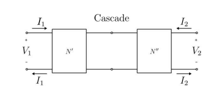


$[h]=[h]'+[h]''$

Cross Product

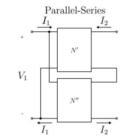
$$[A]\times[B]=\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\times\begin{bmatrix}B_{11} & B_{12}\\ B_{21} & B_{22}\end{bmatrix}=\begin{bmatrix}A_{11}\cdot B_{11}+A_{12}\cdot B_{21} & A_{11}\cdot B_{12}+A_{12}\cdot B_{22}\\ A_{21}\cdot B_{11}+A_{22}\cdot B_{21} & A_{21}\cdot B_{12}+A_{22}\cdot B_{22}\end{bmatrix}$$

Cascade



$[a]=[a]'\times[a]''$ $\quad [b]=[b]''\times[b]'$

Parallel-Series



$[g]=[g]'+[g]''$

Matrix Addition

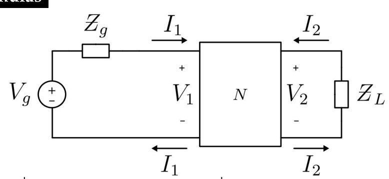
$$[A]+[B]=\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}+\begin{bmatrix}B_{11} & B_{12}\\ B_{21} & B_{22}\end{bmatrix}=\begin{bmatrix}A_{11}+B_{11} & A_{12}+B_{12}\\ A_{21}+B_{21} & A_{22}+B_{22}\end{bmatrix}$$

Parameter Conversion

Determinant: Δ? =?$_{11}$?$_{22}$−?$_{12}$?$_{21}$

z	z_{11}	z_{12}	$\frac{y_{22}}{\Delta y}$	$\frac{-y_{12}}{\Delta y}$	$\frac{\Delta h}{h_{22}}$	$\frac{h_{12}}{h_{22}}$	$\frac{1}{g_{11}}$	$\frac{-g_{12}}{g_{11}}$	$\frac{a_{11}}{a_{21}}$	$\frac{\Delta a}{a_{21}}$	$\frac{b_{22}}{b_{21}}$	$\frac{1}{b_{21}}$
	z_{21}	z_{22}	$-\frac{y_{21}}{\Delta y}$	$\frac{y_{11}}{\Delta y}$	$\frac{-h_{21}}{h_{22}}$	$\frac{1}{h_{22}}$	$\frac{g_{21}}{g_{11}}$	$\frac{\Delta g}{g_{11}}$	$\frac{1}{a_{21}}$	$\frac{a_{22}}{a_{21}}$	$\frac{\Delta b}{b_{21}}$	$\frac{b_{11}}{b_{21}}$
y	$\frac{z_{22}}{\Delta z}$	$\frac{-z_{12}}{\Delta z}$	y_{11}	y_{12}	$\frac{1}{h_{11}}$	$\frac{-h_{12}}{h_{11}}$	$\frac{\Delta g}{g_{22}}$	$\frac{g_{12}}{g_{22}}$	$\frac{a_{22}}{a_{12}}$	$\frac{-\Delta a}{a_{12}}$	$\frac{b_{11}}{b_{12}}$	$\frac{-1}{b_{12}}$
	$\frac{-z_{21}}{\Delta z}$	$\frac{z_{11}}{\Delta z}$	y_{21}	y_{22}	$\frac{h_{21}}{h_{11}}$	$\frac{\Delta h}{h_{11}}$	$\frac{-g_{21}}{g_{22}}$	$\frac{1}{g_{22}}$	$\frac{-1}{a_{12}}$	$\frac{a_{11}}{a_{12}}$	$\frac{-\Delta b}{b_{12}}$	$\frac{b_{22}}{b_{12}}$
h	$\frac{\Delta z}{z_{22}}$	$\frac{z_{12}}{z_{22}}$	$\frac{1}{y_{11}}$	$\frac{-y_{12}}{y_{11}}$	h_{11}	h_{12}	$\frac{g_{22}}{\Delta g}$	$\frac{g_{12}}{\Delta g}$	$\frac{a_{12}}{a_{22}}$	$\frac{\Delta a}{a_{22}}$	$\frac{b_{12}}{b_{11}}$	$\frac{1}{b_{11}}$
	$\frac{-z_{21}}{z_{22}}$	$\frac{1}{z_{22}}$	$\frac{y_{21}}{y_{11}}$	$\frac{\Delta y}{h_{11}}$	h_{21}	h_{22}	$\frac{g_{21}}{\Delta g}$	$\frac{g_{11}}{\Delta g}$	$\frac{-1}{a_{22}}$	$\frac{a_{21}}{a_{22}}$	$\frac{-\Delta b}{b_{11}}$	$\frac{b_{21}}{b_{11}}$
g	$\frac{1}{z_{11}}$	$\frac{-z_{12}}{z_{11}}$	$\frac{\Delta y}{y_{22}}$	$\frac{y_{12}}{y_{22}}$	$\frac{h_{22}}{\Delta h}$	$\frac{-h_{12}}{\Delta h}$	g_{11}	g_{12}	$\frac{a_{21}}{a_{11}}$	$\frac{-\Delta a}{a_{11}}$	$\frac{b_{21}}{b_{22}}$	$\frac{-1}{b_{22}}$
	$\frac{z_{21}}{z_{11}}$	$\frac{\Delta z}{z_{11}}$	$\frac{-y_{21}}{y_{22}}$	$\frac{1}{y_{22}}$	$\frac{-h_{21}}{\Delta h}$	$\frac{h_{11}}{\Delta h}$	g_{21}	g_{22}	$\frac{1}{a_{11}}$	$\frac{a_{12}}{a_{11}}$	$\frac{\Delta b}{b_{22}}$	$\frac{b_{12}}{b_{22}}$
a	$\frac{z_{11}}{z_{21}}$	$\frac{\Delta z}{z_{21}}$	$\frac{-y_{22}}{y_{21}}$	$\frac{-1}{y_{21}}$	$\frac{-\Delta h}{h_{21}}$	$\frac{-h_{11}}{h_{21}}$	$\frac{1}{g_{21}}$	$\frac{g_{22}}{g_{21}}$	a_{11}	a_{12}	$\frac{b_{22}}{\Delta b}$	$\frac{b_{12}}{\Delta b}$
	$\frac{1}{z_{21}}$	$\frac{z_{22}}{z_{21}}$	$\frac{-\Delta y}{y_{21}}$	$\frac{-y_{11}}{y_{21}}$	$\frac{-h_{22}}{h_{21}}$	$\frac{-1}{h_{21}}$	$\frac{g_{11}}{g_{21}}$	$\frac{\Delta g}{g_{21}}$	a_{21}	a_{22}	$\frac{b_{21}}{\Delta b}$	$\frac{b_{11}}{\Delta b}$
b	$\frac{z_{22}}{z_{12}}$	$\frac{\Delta z}{z_{12}}$	$\frac{-y_{11}}{y_{12}}$	$\frac{-1}{y_{12}}$	$\frac{1}{h_{12}}$	$\frac{h_{11}}{h_{12}}$	$\frac{-\Delta g}{g_{12}}$	$\frac{-g_{22}}{g_{12}}$	$\frac{a_{22}}{\Delta a}$	$\frac{a_{12}}{\Delta a}$	b_{11}	b_{12}
	$\frac{1}{z_{12}}$	$\frac{z_{11}}{z_{12}}$	$\frac{-\Delta y}{y_{12}}$	$\frac{y_{22}}{y_{12}}$	$\frac{h_{22}}{h_{12}}$	$\frac{\Delta h}{h_{12}}$	$\frac{-g_{11}}{g_{12}}$	$\frac{-1}{g_{12}}$	$\frac{a_{21}}{\Delta a}$	$\frac{a_{11}}{\Delta a}$	b_{21}	b_{22}

Two Port Formulas



Z_{in}	$z_{11} - \frac{z_{12}z_{21}}{z_{22}+Z_L}$	N/A
	$h_{11} - \frac{h_{12}h_{21}Z_L}{1+h_{22}Z_L}$	N/A
	$\frac{a_{11}Z_L+a_{12}}{a_{21}Z_L+a_{22}}$	$\frac{b_{22}Z_L+b_{12}}{b_{21}Z_L+b_{11}}$
Y_{in}	N/A	$y_{11} - \frac{y_{12}y_{21}Z_L}{1+y_{22}Z_L}$
	N/A	$g_{11} - \frac{g_{12}g_{21}}{g_{22}+Z_L}$
	N/A	N/A
I_2	$\frac{-z_{21}V_g}{(z_{11}+Z_g)(z_{22}+Z_L)-z_{12}z_{21}}$	$\frac{y_{21}V_g}{1+y_{22}Z_g+\Delta y Z_g Z_L}$
	$\frac{h_{21}V_g}{(1+h_{22}Z_L)(h_{11}+Z_g)-h_{12}h_{21}Z_L}$	$\frac{-g_{21}V_g}{(1+g_{11}Z_g)(g_{22}+Z_L)-g_{12}g_{21}Z_g}$
	$\frac{-V_g}{a_{11}Z_L+a_{12}+a_{21}Z_gZ_L+a_{22}Z_g}$	$\frac{-V_g\Delta b}{b_{11}Z_g+b_{21}Z_gZ_L+b_{22}Z_L+b_{12}}$
V_{Th}	$\frac{z_{21}}{z_{11}+Z_g}V_g$	$\frac{-y_{21}V_g}{y_{22}+\Delta y Z_g}$
	$\frac{-h_{21}V_g}{h_{22}Z_g+\Delta h}$	$\frac{g_{21}V_g}{1+g_{11}Z_g}$
	$\frac{V_g}{a_{11}+a_{21}Z_g}$	$\frac{V_g\Delta b}{b_{22}+b_{21}Z_g}$

Two Port Formulas Cont'd

Z_{Th}	$z_{22} - \frac{z_{12}z_{21}}{z_{11}+Z_g}$	$\frac{1+y_{11}Z_g}{y_{22}+\Delta y Z_g}$
	$\frac{Z_g+h_{11}}{h_{22}Z_g+\Delta h}$	$g_{22} - \frac{g_{12}g_{21}Z_g}{1+g_{11}Z_g}$
	$\frac{a_{12}+a_{22}Z_g}{a_{11}+a_{21}Z_g}$	$\frac{b_{11}Z_g+b_{12}}{b_{21}Z_g+b_{22}}$
$\frac{I_2}{I_1}$	$\frac{-z_{21}}{z_{22}+Z_L}$	$\frac{y_{21}}{y_{11}+\Delta y Z_L}$
	$\frac{h_{21}}{1+h_{22}Z_L}$	$\frac{-g_{21}}{g_{11}Z_L+\Delta g}$
	$\frac{-1}{a_{21}Z_L+a_{22}}$	$\frac{-\Delta b}{b_{11}+b_{21}Z_L}$
$\frac{V_2}{V_1}$	$\frac{z_{21}Z_L}{z_{11}Z_L+\Delta z}$	$\frac{-y_{21}Z_L}{1+y_{22}Z_L}$
	$\frac{-h_{21}Z_L}{\Delta h Z_L+h_{11}}$	$\frac{g_{21}Z_L}{g_{22}+Z_L}$
	$\frac{Z_L}{a_{11}Z_L+a_{12}}$	$\frac{\Delta b Z_L}{b_{12}+b_{22}Z_L}$
$\frac{V_2}{V_g}$	$\frac{z_{21}Z_L}{(z_{11}+Z_g)(z_{22}+Z_L)-z_{12}z_{21}}$	$\frac{y_{21}Z_L}{y_{12}y_{21}Z_gZ_L-(1+y_{11}Z_g)(1+y_{22}Z_L)}$
	$\frac{-h_{21}Z_L}{(h_{11}+z_g)(1+h_{22}Z_L)-h_{12}h_{21}Z_L}$	$\frac{g_{21}Z_L}{(1+g_{11}Z_g)(g_{22}+Z_L)-g_{12}g_{21}Z_g}$
	$\frac{Z_L}{(a_{11}+a_{21}Z_g)Z_L+a_{12}+a_{22}Z_g}$	$\frac{\Delta b Z_L}{b_{12}+b_{11}Z_g+b_{22}Z_L+b_{21}Z_gZ_L}$

SI

Base Units

Quantity	Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Thermodynamic Temperature	kelvin	K
Amount of substance	mole	mol
Luminous Intensity	candela	cd

Derived Units

Quantity	Unit (Symbol)	Formula
Frequency	hertz (Hz)	s^{-1}
Force	newton (N)	$kg \cdot m/s^2$
Energy or work	joule (J)	$N \cdot m$
Power	watt (W)	J/s
Electric Charge	coulomb (C)	$A \cdot s$
Electric potential	volt (V)	J/C
Electrical Resistance	ohm (Ω)	V/A
Electrical Conductance	siemens (S)	A/V
Electrical Capacitance	farad (F)	C/V
Magnetic Flux	weber (Wb)	$V \cdot s$
Inductance	henry (H)	Wb/A

Prefixes

Prefix		Power
zetta-	(Z)	10^{21}
exa-	(E)	10^{18}
peta-	(P)	10^{15}
tera-	(T)	10^{12}
giga-	(G)	10^{9}
mega-	(M)	10^{6}
kilo-	(k)	10^{3}
hecto-	(h)	10^{2}
deka-	(da)	10^{1}
– Base –		
deci-	(d)	10^{-1}
centi-	(c)	10^{-2}
milli-	(m)	10^{-3}
micro-	(μ)	10^{-6}
nano-	(n)	10^{-9}
pico-	(p)	10^{-12}
fento-	(f)	10^{-15}
atto-	(a)	10^{-18}
zepto-	(z)	10^{-21}

♡ Made by Kiva M. ♡ Check out more of my work at kivamccr.xyz ♡