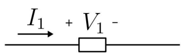
Linear Circuits I [Universal Method]

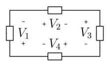
THIS IS NOT OFFICIAL, USE AT YOUR OWN RISK
—— ITEMS ARE NOT IN ORDER ——

Passive Sign Convention



Kirchhoff's Laws

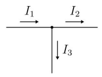
KVL



$V_1 + V_2 + V_3 + V_4 = 0$

The sum of voltage sources and drops around a closed loop must be zero.

KCL



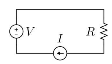
$-I_1 + I_2 + I_3 = 0$

In (-) Out (+)

The sum of currents entering and leaving a node must be zero.

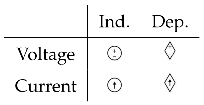
Ohm's Law & Power

Ohm's Law $V = IR$ Power $P = IV$ Absorbing (+) Generating (-)

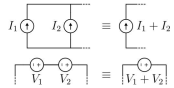


Sources

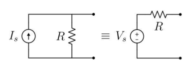
Sources



Addition/Subtraction



Transformation

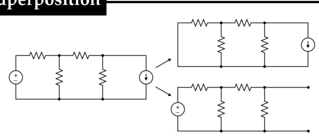


Resistors

The element with resistance R, measured in ohms (Ω).

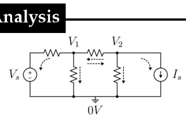
Ohm's Law	$V = IR$
Power	$P = IV$
Series Equivalents	$R_{eq} = R_1 + R_2 + ... + R_n$
Parallel Equivalents	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n}$ or $R_{eq\{R_1,R_2\}} = \frac{R_1 R_2}{R_1+R_2}$
Voltage Division (Series)	$V_{\text{want}} = \frac{R_{\text{want}}}{R_1+R_2+...+R_n} \cdot V_{\text{source}} = \frac{R_{\text{want}}}{R_{eq}} \cdot V_{\text{source}}$
Current Division (Parallel)	$I_{\text{want}} = \frac{\left(\frac{1}{R_1}+\frac{1}{R_2}+...+\frac{1}{R_n}\right)^{-1}}{R_{\text{want}}} \cdot I_{\text{source}} = \frac{R_{eq}}{R_{\text{want}}} \cdot I_{\text{source}}$

Superposition



When there is more than one independent source of energy or in the case of AC more than one frequency, the response can be evaluated separately and added together.

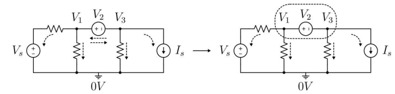
Node Voltage Analysis



$$\frac{V_1 - V_s}{R} + \frac{V_1 - 0}{R} + \frac{V_1 - V_2}{R} = 0$$
$$\frac{V_2 - V_1}{R} + \frac{V_2 - 0}{R} + I_s = 0$$

Create a system of equations using the KCL of two or more nodes, to find the node voltage. The number of KCLs required is the number of nodes (remember to include the ground) minus 1.

Supernode

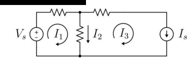


$(V_1 - V_3 = V_2)$

$$\frac{V_1 - V_s}{R} + \frac{V_1 - 0}{R} + \frac{V_3 - 0}{R} + I_s = 0$$

When there is a voltage source between two nodes, we can create an equation for their relationship. Allowing a single KCL for two nodes.

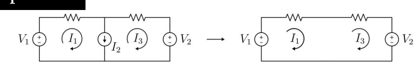
Mesh Current Analysis



$-V_s + I_1 R + I_2 R = 0$ $I_3 = I_s$
$I_3 R + V_s - I_2 R = 0$ $I_2 = I_1 - I_3$

Create a system of equations using the KVL of two or more meshes to find the current of each mesh. On any segments that share two meshes the current in that segment is found using a system of equations.

Supermesh

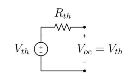


$-V_1 + I_1 R + I_3 R + V_2 = 0$
$I_1 = I_2 - I_3$

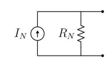
When there is a current source between two meshes, we can create an equation for their relationship. Allowing a single KVL for two meshes.

Thevenin & Norton Equivalence

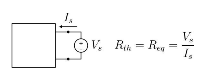
V_{th} — To find V_{th}, open circuit the terminals, and then solve for the voltage across the terminals.



I_N — To find I_N, short the terminals, and then solve for the current through the terminals.

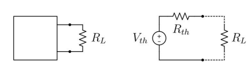


R_{th} R_N — To find R_{th} or R_N, set all independent sources accordingly, then you can either simplify down the resistors or alternatively add a test voltage and current to the segment and divide the test voltage by the test current.



Maximum Power Transfer

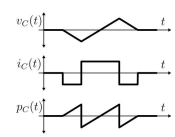
Maximum power transfer occurs when $R_L = R_{th}$, and can be calculated via $p_{max} = \frac{V_{th}^2}{4R_L}$



Capacitors

The element with capacitance C, measured in farads (F). If voltage is constant (DC), the current is zero aka open circuit. Voltage cannot change instantly.
Full → Open & Empty → Short.

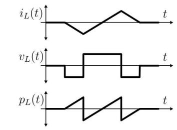
Definition	$C = \frac{q}{v}$
Voltage	$v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i(t)d\tau$
Current	$i(t) = C\frac{dv}{dt}$
Power	$p(t) = C\frac{dv}{dt} \cdot v(t)$
Energy	$\omega = \frac{1}{2}Cv^2$
Series Equivalents	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_n}$ or $C_{eq\{C_1,C_2\}} = \frac{C_1 C_2}{C_1+C_2}$
Parallel Equivalents	$C_{eq} = C_1 + C_2 + ... + C_n$
Voltage Division (Series)	$v_{\text{want}} = \frac{\left(\frac{1}{C_1}+\frac{1}{C_2}+...+\frac{1}{C_n}\right)^{-1}}{C_{\text{want}}} \cdot v_{\text{source}} = \frac{C_{eq}}{C_{\text{want}}} \cdot v_{\text{source}}$
Current Division (Parallel)	$i_{\text{want}} = \frac{C_{\text{want}}}{C_1+C_2+...+C_n} \cdot i_{\text{source}} = \frac{C_{\text{want}}}{C_{eq}} \cdot i_{\text{source}}$



Inductors

The element with inductance L, measured in henrys (H). If current is constant (DC), the voltage is zero aka short circuit. Current cannot change instantly.
Full → Short & Empty → Open

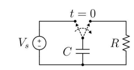
Voltage	$v(t) = L\frac{di}{dt}$
Current	$i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v(t)d\tau$
Power	$p(t) = i(t) \cdot L\frac{di}{dt}$
Energy	$\omega = \frac{1}{2}Li^2$
Series Equivalents	$L_{eq} = L_1 + L_2 + ... + L_n$
Parallel Equivalents	$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + ... + \frac{1}{L_n}$ or $L_{eq\{L_1,L_2\}} = \frac{L_1 L_2}{L_1+L_2}$
Voltage Division (Series)	$v_{\text{want}} = \frac{L_{\text{want}}}{L_1+L_2+...+L_n} \cdot v_{\text{source}} = \frac{L_{\text{want}}}{L_{eq}} \cdot v_{\text{source}}$
Current Division (Parallel)	$i_{\text{want}} = \frac{\left(\frac{1}{L_1}+\frac{1}{L_2}+...+\frac{1}{L_n}\right)^{-1}}{L_{\text{want}}} \cdot i_{\text{source}} = \frac{L_{eq}}{L_{\text{want}}} \cdot i_{\text{source}}$



Natural Response

The response of a Capacitor or Inductor when Sources are removed from the RC/RL Circuit.

RC

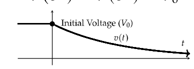


Switch is closed a long time
Energy Stored
Capacitor → Open Circuit
Switch Opens
Open Voltage → Voltage Source
Evaluate

$v(t) = V_0 e^{-\frac{t}{\tau}},\ t \geq 0$

$\tau = C \cdot R$

$V(0^-) = V(0^+) = V_0$

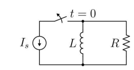


$i(t) = \frac{1}{R}V_0 e^{-\frac{t}{\tau}},\ t \geq 0^+$

$p(t) = i(t) \cdot v(t),\ t \geq 0^+$

$\omega(t) = \frac{1}{2} \cdot C \cdot v(t)^2,\ t \geq 0$

RL

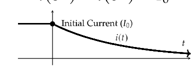


Switch is closed a long time
Energy Stored
Inductor → Short Circuit
Switch Opens
Short Current → Current Source
Evaluate

$i(t) = I_0 e^{-\frac{t}{\tau}},\ t \geq 0$

$\tau = \frac{L}{R}$

$i(0^-) = i(0^+) = I_0$



$v(t) = I_0 R e^{-\frac{t}{\tau}},\ t \geq 0^+$

$p(t) = i(t) \cdot v(t),\ t \geq 0^+$

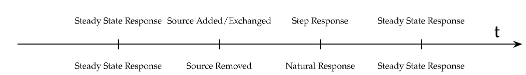
$\omega(t) = \frac{1}{2} \cdot L \cdot i(t)^2,\ t \geq 0$

Steady state will be reached at $\approx \tau = 5t$

Steady State Response

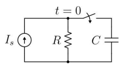
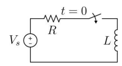
1. Convert all Inductors, Resistors, and Capacitors to Impedance.
2. Use Superposition if Sources have differing Frequencies (ω).
3. Set up a KVL, KCL, Nodal, Mesh, etc...
4. Solve using Complex Number properties.

RC/RL Response Big Picture



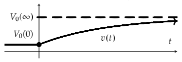
Step Response

The response of a Capacitor or Inductor when Sources are added or exchanged to/in the RC/RL Circuit.

RC	RL
	
Switch is open a long time	Switch is open a long time
Draw circuit at $t < 0$	Draw circuit at $t < 0$
Capacitor → Open Circuit	Inductor → Short Circuit
Solve for $i_0(0^-)$ & $V_0(0^-)$	Solve for $i_0(0^-)$ & $V_0(0^-)$
Draw circuit at $t = 0^+$	Draw circuit at $t = 0^+$
Carry $V_0(0^-)$ as Source $V_0(0^+)$	Carry $i_0(0^-)$ as Source $i_0(0^+)$
Solve for $i_0(0^+)$	Solve for $V_0(0^+)$
Draw circuit at $t = \infty$	Draw circuit at $t = \infty$
Capacitor → Open Circuit	Inductor → Short Circuit
Solve for $i_0(\infty)$ & $V_0(\infty)$	Solve for $i_0(\infty)$ & $V_0(\infty)$
Plug into $V_0(t)$ & $i_0(t)$	Plug into $V_0(t)$ & $i_0(t)$
Solve for τ	Solve for τ

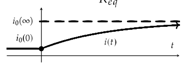
RC:

$$V_0(t) = V_0(\infty) + [V_0(0^+) - V_0(\infty)]\, e^{-\frac{t}{\tau}}$$
$$i_0(t) = i_0(\infty) + [i_0(0^+) - i_0(\infty)]\, e^{-\frac{t}{\tau}}$$
$$\tau = C \cdot R_{eq}$$



RL:

$$V_0(t) = V_0(\infty) + [V_0(0^+) - V_0(\infty)]\, e^{-\frac{t}{\tau}}$$
$$i_0(t) = i_0(\infty) + [i_0(0^+) - i_0(\infty)]\, e^{-\frac{t}{\tau}}$$
$$\tau = \frac{L}{R_{eq}}$$



Steady state will be reached at $\approx \tau = 5t$

Complex Sources

$$\underbrace{v_s(t)}_{\text{Source}} = \underbrace{V_m}_{\text{Amplitude}} \cos(\underbrace{\omega t}_{\text{Frequency }(\omega)} + \underbrace{\theta_v}_{\text{(Voltage) Phase }(\theta)}) \rightarrow V_s = V_m \angle \theta_v = V_m e^{j\theta_v}$$

$$i_s(t) = I_m \cos(\omega t + \theta_i) \rightarrow I_s = I_m \angle \theta_i = I_m e^{j\theta_i}$$

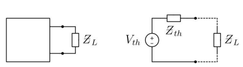
Root Mean Square (RMS): The DC current/voltage that dissipates the same amount of power as the average power of the AC equivalent.

For a Sinusoidal Function: $V_{rms} = \frac{V_m}{\sqrt{2}}$ $\quad I_{rms} = \frac{I_m}{\sqrt{2}}$

Maximum Power Transfer [Complex]

Maximum power transfer occurs when $Z_L = Z_{th}^*$, and can be calculated via

$p_{max} = \frac{|V_{th,rms}|^2}{4R_{th}} = \frac{|V_{th,max}|^2}{8R_{th}}$



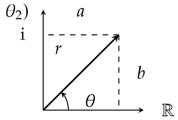
$Z_{th} = R_{Th} + jX_{th}$ $\quad Z_L = R_L + jX_L$

Complex Numbers

$$j = \sqrt{-1}$$

Form	Equation	Conversion
Rectangular	$a + jb$	To polar: $r = \sqrt{a^2 + b^2}$, $\angle\theta = \tan^{-1}(\frac{b}{a})$
Polar	$re^{j\theta} = r\angle\theta$	To rectangular: $a = rcos(\theta)$, $b = rsin(\theta)$

Operation	Property
Addition	$(a_1 + jb_1) + (a_2 + jb_2) = (a_1 + a_2) + j(b_1 + b_2)$
Subtraction	$(a_1 + jb_1) - (a_2 + jb_2) = (a_1 - a_2) + j(b_1 - b_2)$
Multiplication	$r_1\angle\theta_1 \cdot r_2\angle\theta_2 = (r_1 \cdot r_2)\angle(\theta_1 + \theta_2)$
Division	$\frac{r_1\angle\theta_1}{r_2\angle\theta_2} = \frac{r_1}{r_2}\angle(\theta_1 - \theta_2)$



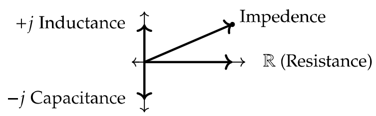
deg	0°	30°	45°	60°	90°
rad	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$$

$$1 + j = \sqrt{2}\angle 45°$$
$$\{a + jb\}^* = a - jb$$
$$\frac{1}{j} = -j$$

Euler's Formula: $e^{j\theta} = \cos\theta + j\sin\theta$

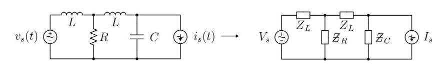
Impedance

Impedance is symbolized by Z and is measured in ohms (Ω).



$Z_R = R$ $\quad Z_L = j\omega L$ $\quad Z_C = \frac{1}{j\omega C} = -\frac{j}{\omega C}$

Impedance allows us to evaluate the response of Inductors, Resistors, and Capacitors all at once by evaluating them in the Phasor domain.



Ohm's Law	$V = IZ$
Series Equivalents	$Z_{eq} = Z_1 + Z_2 + \ldots + Z_n$
Parallel Equivalents	$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \ldots + \frac{1}{Z_n}$
Voltage Division (Series)	$V_{\text{want}} = \frac{Z_{\text{want}}}{Z_1 + Z_2 + \ldots + Z_n} \cdot V_{\text{source}} = \frac{Z_{\text{want}}}{Z_{eq}} \cdot V_{\text{source}}$
Current Division (Parallel)	$I_{\text{want}} = \frac{\left(\frac{1}{Z_1} + \frac{1}{Z_2} + \ldots + \frac{1}{Z_n}\right)^{-1}}{Z_{\text{want}}} \cdot I_{\text{source}} = \frac{Z_{eq}}{Z_{\text{want}}} \cdot I_{\text{source}}$

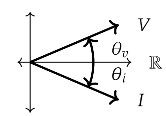
Complex & Apparent Power

Average (Real) Power (W): $P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)$

Instantaneous (Reactive) Power (VAR): $Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)$

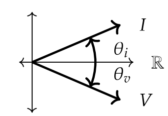
Circuit	Power	Power Factor Angle
Purely Resistive:	$p(t) = P + P\cos(2\omega t)$	$\theta_v - \theta_i = 0°$
Purely Inductive:	$p(t) = -Q\sin(2\omega t)$	$\theta_v - \theta_i = 90°$
Purely Capacitive:	$p(t) = -Q\sin(2\omega t)$	$\theta_v - \theta_i = -90°$

Lagging PF



Current Lags Voltage (Inductive)

Leading PF



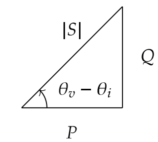
Current Leads Voltage (Capacitive)

Complex Power (VA):

- $S = P + jQ$
- $S = V_{rms}e^{j\theta_v} \cdot I_{rms}e^{-j\theta_i}$
- $S = V_{rms} \cdot I_{rms}^*$

Apparent Power (VA): $|S| = \sqrt{P^2 + Q^2}$

Power Factor $= \cos(\theta_v - \theta_i) = \frac{\text{Real Power}}{\text{Apparent Power}}$



SI

Base Units

Quantity	Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Thermodynamic Temperature	kelvin	K
Amount of substance	mole	mol
Luminous Intensity	candela	cd

Derived Units

Quantity	Unit (Symbol)	Formula
Frequency	hertz (Hz)	s^{-1}
Force	newton (N)	$kg \cdot m/s^2$
Energy or work	joule (J)	$N \cdot m$
Power	watt (W)	J/s
Electric Charge	coulomb (C)	$A \cdot s$
Electric potential	volt (V)	J/C
Electrical Resistance	ohm (Ω)	V/A
Electrical Conductance	siemens (S)	A/V
Electrical Capacitance	farad (F)	C/V
Magnetic Flux	weber (Wb)	$V \cdot s$
Inductance	henry (H)	Wb/A

Prefixes

Prefix		Power
zetta-	(Z)	10^{21}
exa-	(E)	10^{18}
peta-	(P)	10^{15}
tera-	(T)	10^{12}
giga-	(G)	10^{9}
mega-	(M)	10^{6}
kilo-	(k)	10^{3}
hecto-	(h)	10^{2}
deka-	(da)	10^{1}
– Base –		
deci-	(d)	10^{-1}
centi-	(c)	10^{-2}
milli-	(m)	10^{-3}
micro-	(μ)	10^{-6}
nano-	(n)	10^{-9}
pico-	(p)	10^{-12}
fento-	(f)	10^{-15}
atto-	(a)	10^{-18}
zepto-	(z)	10^{-21}