UCF
ODE
– EXAM 1 NOTES

The following topics are covered:

- ☐ 1.1 Differential Equations
- ☐ 1.2 Solutions & Initial Value Problems
- ☐ 1.3 Direction Fields
- ☐ 2.2 Separable Equations
- ☐ 2.3 Linear Equations
- ☐ 2.4 Exact Equations

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Differential Equations:

- Ordinary: A function involving only ordinary derivatives with respect to a single independent variable.
  - Linear Ordinary: A function whose derivatives appear linearly,
    $a_n(x)\frac{d^ny}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_1(x)\frac{dy}{dx} + a_0(x)y = F(x)$.
  - Non-Linear Ordinary: A function whose derivatives do not appear linearly.
- Partial: A function involving partial derivatives with respect to more than one independent variable.

Conditions for linearity:

(i) Degree of the differential equation is 1.
$\left(\frac{d^3y}{dx^3}\right)^3 - 5\left(\frac{d^2y}{dx^2}\right)^2 - 3y = 0 \therefore \oslash$ linear

(ii) Exponent of each differential quotient of the differential equation is 1.
$\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 + 7y = x \therefore \oslash$ linear

(iii) Exponent of each dependent variable, y, of the differential equation is 1.
$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y^2 = x \therefore \oslash$ linear

(iv) No term contains product of the dependent variable and its differential coefficient.
$x^2 + y - 2xy\frac{dy}{dx} = 0 \therefore \oslash$ linear

Orders:
$1^{st} = y' + \cdots = \frac{dy}{dx} + \cdots$
$2^{nd} = y'' + \cdots = \frac{d^2y}{dx^2} + \cdots$
$3^{rd} = y''' + \cdots = \frac{d^3y}{dx^3} + \cdots$
etc...

Independent & Dependent:
A differential equation describes the dependent variable in terms of the independent variable(s).
$\frac{d^n(dependent)}{d(independant)^n}$

Solutions & Initial Value Problems:
The solution is the $f(x)$ that makes $F(t, f(x), f'(x), \cdots, f^n(x)) = 0$ true for all values.

To solve an Initial Value Problem (IVP), begin by finding the explicit solution and then utilize the initial condition $y(x_0) = y_0$ to determine the constant of integration, denoted as C.

Existence and Uniqueness Theorem:
For the initial value problem $\frac{dy}{dx} = f(x, y),\ y(x_0) = y_0$. If f & $\frac{df}{dy}$ are continuous functions in some rectangle/domain $\mathbb{R} = \{(x, y) : a < x < b,\ c < y < d\}$ that contains a point (x_0, y_0) then there exists a unique solution.

Explicit vs. Implicit Solutions:

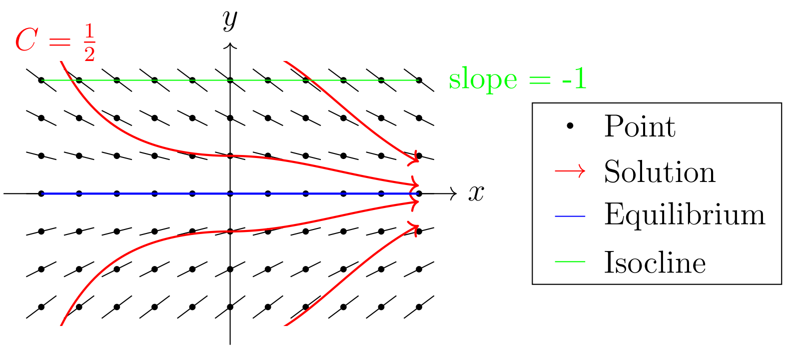
- An explicit solution defines the dependant variable in terms of the independent, ex. $y = \sqrt{25 - x^2}$
- An implicit solution has the independent and dependent variables intertwined so the relationship is not immediately obvious, ex. $y^2 + x^2 = 25$

Direction Fields: A plot of short line segments depicting the slope of the solution curve at each point in a domain.

$> 0 = positive$
$< 0 = negative$
$\infty = vertical$
$0 = horizontal$

A direction will flip when crossing the equilibrium, however the equilibrium cannot be crossed by a solution.

Isoclines: A line which joins neighbouring points with the same gradient.



Separable Equations:
For the separable equation: $\frac{dy}{dx} = g(x)p(y)$

- Move the independent and dependant variables to their respective sides "separate them", $\left(\frac{dy}{dx} = g(x)p(y)\right) \times \frac{dx}{p(y)} \equiv h(y)dy = g(x)dx$
- Integrate both sides respectively $\int h(y)dy = \int g(x)dx \equiv H(y) + C_1 = G(x) + C_2$
- Combine the resulting constants of integration $H(y) = G(x) + C$

ex.

- ▷ Find the explicit solution of $\frac{x^2}{y} = 4y\frac{dy}{dx}$
- ▷ Move to respective sides $x^2 dx = 4y^2 dy$
- ▷ Integrate respectively $\int x^2 dx = \int 4y^2 dy \equiv \frac{x^3}{3} + C_1 = \frac{4y^3}{3} + C_2$
- ▷ Combine constants of integration $\frac{4y^3}{3} = \frac{x^3}{3} + C$

▷ Solve for y, $y = \sqrt[3]{\frac{x^3}{4}}$

Linear Equations:
For the linear equation: $\frac{dy}{dx} + P(x)y = Q(x)$

- Calculate the integration factor $\mu(x) = e^{\int P(x)dx}$
- Multiply both sides of the linear equation by the integration factor, $\mu(x)\frac{dy}{dx} + \mu(x)P(x)y = \mu(x)Q(x)$
- The left side will simplify to $\frac{d}{dx}\left[\mu(x)y\right] = \mu(x)Q(x)$
- Integrate both sides respectively $\mu(x)y = \int \mu(x)Q(x))dx + C$
- Divide by the integrating factor to solve for y, $y = \frac{1}{\mu(x)}\left(\int \mu(x)Q(x))dx + C\right)$

ex.

▷ Find the general solution to $\frac{dy}{dx} = \frac{y}{x} + 5x + 3$

▷ Rearrange the equation into linear format $\frac{dy}{dx} - \frac{1}{x}y = 5x + 3$

▷ Calculate the integrating factor $\mu(x) = e^{\int \left(-\frac{1}{x}\right)dx} = e^{-\int \left(\frac{1}{x}\right)dx} = e^{-\ln|x|} = e^{\ln\left|\frac{1}{x}\right|} = \frac{1}{x}$

▷ Multiply both sides by the integration factor $\frac{1}{x}\left(\frac{dy}{dx} - \frac{1}{x}y\right) = 5x \times \frac{1}{x} + 3 \times \frac{1}{x}$

▷ Simplify $\frac{d}{dx}\left[\frac{1}{x}y\right] = 5 + \frac{3}{x}$

▷ Integrate with respect to x, $\frac{1}{x}y = \int \left(5 + \frac{3}{x}\right)dx = 5x + 3\ln|x| + C$

▷ Divide by the integration factor to solve for y, $y = 5x^2 + 3x\ln|x| + Cx$

Integrating Factors Proof:
For the linear equation: $y' + P(x)y = Q(x)$

- To solve for the integrating factor $\mu(x)$
- Multiplying the integration factor across the base equation yields $\mu(x)y' + \mu(x)P(x)y = \mu(x)Q(x)$
- Our end goal is to integrate $\frac{d}{dx}\left[\mu(x)y\right] = \mu(x)Q(x)$
- Recognize that this is product rule
- Therefore the following must be true $\mu'(x) = \mu(x)P(x)$
- Separate and integrate $\int \frac{\mu'(x)}{\mu(x)}dx = \int P(x)dx \equiv \ln(\mu(x)) = \int P(x)dx$
- Raise both sides to powers of e, $\mu(x) = e^{\int P(x)dx}$

$*$ Sorry for the change in notation above.

Exact Equations:
The equation $M(x,y)dx + N(x,y)dy = 0$ is exact if $\frac{dM}{dy} = \frac{dN}{dx}$

If the equation is exact there exists a function $F(x,y)$ that satisfies $\frac{dF}{dx} = M$ & $\frac{dF}{dy} = N$

- Integrate the first equation with respect to x, $F(x,y) = \int M(x,y)dx + g(y)$
- Differentiate both sides with respect to y, $\frac{dF}{dy}(x,y) = \frac{d}{dy}\int M(x,y)dx + g'(y)$
- Substitute since $N(x,y) := \frac{dF}{dy}$ and solve for $g'(y)$,
  $g'(y) = N(x,y) - \frac{d}{dy}\int M(x,y)dx$
- Integrate $g(y)$, and substitute into the initial equation to obtain $F(x,y)$.

ex.

- ▷ Find the implicit solution for $(4x^3y+2)\,dx + (x^4-3)\,dy = 0$
- ▷ Test for exactness by deriving respectively
  $\begin{cases} \frac{d}{dy}(4x^3y+2) = 4x^3 \\ \frac{d}{dx}(x^4-3) = 4x^3 \\ 4x^3 = 4x^3 \therefore \checkmark \text{ Exact} \end{cases}$
- ▷ Integrate $M(x,y)$ with respect to x, $\int(4x^3y+2)\,dx \to F(x,y) = x^4y + 2x + g(y)$
- ▷ Differentiate both sides with respect to y, $\frac{dF}{dy} = x^4 + g'(y)$
- ▷ Substitute and solve for $g'(y)$, $g'(y) = N(x,y) - x^4 = (x^4-3) - x^4 = -3$
- ▷ Integrate $g(y)$, $\int -3dy = -3y$
- ▷ Substitute $g(y)$ into the initial equation, $F(x,y) = x^4y + 2x - 3y$

Alternatively

- Integrate $\int M(x,y)dx$ & $\int N(x,y)dy$ respectively
- Find the the union $F(x,y) = \int M(x,y)dx \cup \int N(x,y)dy$

ex.

- ▷ Find the implicit solution for $(4x^3y+2)\,dx + (x^4-3)\,dy = 0$
- ▷ Integrate $\int M(x,y)dx$ & $\int N(x,y)dy$ respectively,
  $\begin{cases} \int(4x^3y+2)\,dx = x^4y + 2x \\ \int(x^4-3)\,dy = x^4y - 3y \end{cases}$
- ▷ Find the union $(x^4y+2x) \cup (x^4y-3y) = x^4y + 2x - 3y$

$*$ If the equation is not exact, refer to Exam 2 Notes § Special Integrating Factors

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– EXAM 2 NOTES

The following topics are covered:

- ☐ 2.5 Special Integrating Factors
- ☐ 2.6 Substitutions & Transformations
- ☐ 4.1 Mass-Spring Oscillator
- ☐ 4.2 Homogeneous Linear ODE of 2^{nd} Order
- ☐ 4.3 Complex Roots
- ☐ 4.4 Method of Undetermined Coefficients
- ☐ 4.5 Superposition Principle

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Special Integrating Factors:
An integrating factor is an equation $\mu(x, y)$ that when multiplied against makes an equation exact.

To Find the integrating factor:

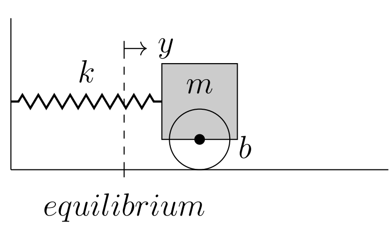
- Identify the form $M(x, y)dx + N(x, y)dy = 0$
- If $\frac{dM}{dy} \neq \frac{dN}{dx}$ the equation is not exact an integrating factor must be found.
- Find the integrating factor option whose integral contains only one variable.
  - $\mu(x) = e^{\int \frac{\frac{dM}{dy} - \frac{dN}{dx}}{N} dx}$ or $\mu(y) = e^{\int \frac{\frac{dN}{dx} - \frac{dM}{dy}}{M} dy}$
- Multiply the integrating factor against the equation, resulting in $\mu(x, y)M(x, y)dx + \mu(x, y)N(x, y)dy = 0$
- Now solve the newly exact equation.

Substitutions & Transformations:
If an equation is not separable, exact, or linear a substitution or transformation, can be applied to make the equation solvable under our current methods.

For a homogeneous equation $\frac{dy}{dx} = F(x, y)$:

- If $F(x, y)$ is in terms of a variable combination such as $\frac{y}{x}$
- Rewrite the equation having substituted v for the variable combo.
- Identify the new equation as exact, linear, or separable and solve.
- Convert the solution back to its original terms.

Mass-Spring Oscillator:



There are the following components:

- k = spring constant
- m = mass
- b = damping (ex. friction)
- y = displacement

There are the following forces:

- Newton's 2^{nd} Law $F = my''$
- Hooke's Law $F_{spring} = -ky$
- Friction is proportional to velocity so:
  $F_{friction} = -by'$
- External forces lumped together as $F_{ext}(t)$

Combining these forces creates the equation: $my'' = -ky - by' + F_{ext}(t)$. Rearranging the equation results in $my'' + by' + ky = F_{ext}(t)$. This reassembles an auxiliary equation and can be solved using the techniques below.

Homogeneous Linear ODE of 2^{nd} Order:
For the auxiliary equation: $ay'' + by' + cy = f(t)$

If $f(t) = 0$ aka homogeneous

- Convert $ay'' + by' + cy$ to the form $ar^2 + br + c$
- Factor for the roots r_1 & r_2
- If $r_1 \neq r_2$: general solution $= C_1e^{r_1t} + C_2e^{r_2t}$
- If $r_1 = r_2$: general solution $= C_1e^{r_1t} + C_2te^{r_2t}$
- For an initial value problem, solve for C_1 & C_2, by plugging,
  y_h into the auxiliary equation at the respective derivatives.

Complex Roots:
For the auxiliary equation: $ay'' + by' + cy = f(t)$

If $f(t) = 0$ & roots are complex

- Convert $ay'' + by' + cy$ to the form $ar^2 + br + c$
- Factor for the complex roots r_1 & r_2
- If r_1 & r_2 are the form $\alpha \pm \beta$: general solution $= C_1e^{r_1t} + C_2e^{r_2t}$
- If r_1 & r_2 are the form $\alpha \pm \beta i$: general solution $= C_1e^{\alpha t}cos(\beta t) + C_2e^{\alpha t}sin(\beta t)$
- For an initial value problem, solve for C_1 & C_2, by plugging,
  y_h into the auxiliary equation at the respective derivatives.

Method of Undetermined Coefficients:
For the auxiliary equation: $ay'' + by' + cy = f(t)$

If $f(t) \neq 0$ aka non-homogeneous

- Apply the above methods to find the homogeneous solution(y_h), do not solve for C_1 or C_2 yet.
- Choose a y_p aka particular solution to test based on $f(t)$. Examples are in the chart below.
- If $f(t)$ contains a root then y_pt, if two roots y_pt^2.
- Plug y_p into the into the auxiliary equation at the respective derivatives.
- Solve for constants, ex. A, B, C...
- For an initial value problem, solve for C_1 & C_2, by plugging, the general solution $y_h + y_p$ into the auxiliary equation at the respective derivatives.

$f(t)$	y_p
7	A
t	$At + B$
$t + 2$	$At + B$
t^2	$At^2 + Bt + C$
t^2e^{2t}	$(At^2 + Bt + C)e^{2t}$
$tsin(t) + 7$	$(At + B)sin(t) + (Ct + D)cos(t) + E$

Superposition Principle:
For the auxiliary equation: $ay'' + by' + cy = f(t)$

If $f(t)$ are two distinct functions $y_1 + y_2$, y_p can be solved seperately as y_{p1} & y_{p2}.
As a result the general equation is $y_h + y_{p1} + y_{p2}$

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– EXAM 3 NOTES

The following topics are covered:

- ☐ 4.6 Variation of Parameters
- ☐ 4.7 Variable Coefficient Equations
- ☐ 6.1 Basic Theory of Linear Differential Equations
- ☐ 6.2 Homogeneous Linear Equations with Constant Coefficients
- ☐ 6.3 Undetermined Coefficients and the Annihilator Method
- ☐ 6.4 Method of Variation of Parameters

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Variation of Parameters (2^{nd} Order):
For the auxiliary equation: $ay'' + by' + cy = f(t)$

If $f(t) \neq 0$ aka non-homogeneous

- Find $y_1(t)$ & $y_2(t)$ via the homogeneous solution $y_h(t) = C_1y_1(t) + C_2y_2(t)$.
- Calculate the Wronskian $W = \begin{vmatrix} y_1 & y_2 \\ {y_1}' & {y_2}' \end{vmatrix} = y_1{y_2}' - {y_1}'y_2$
- Find the particular solution $y_p(t) = v_1y_1(t) + v_2y_2(t)$,
  $v_1(t) = \int \frac{-f(t)y_2(t)}{W(y_1,y_2)}dt$ & $v_2(t) = \int \frac{f(t)y_1(t)}{W(y_1,y_2)}dt$.
- The general solution is $y_h + y_p$.
- For an initial value problem, solve for C_1 & C_2, by plugging, the general solution $y_h + y_p$ into the auxiliary equation at the respective derivatives.

ex.

- ▷ Find the general solution of $y'' - 3y' + 2y = e^{-t}$
- ▷ Substitute $r^2 - 3r + 2 = 0$ for $y'' - 3y' + 2y = e^{-t}$ to find homogeneous solution (y_h)
- ▷ $(r-2)(r-1)$, therefore roots are 1 & 2
- ▷ Homogeneous solution $(y_h) = C_1e^{1t} + C_2e^{2t}$, therefore $y_1 = e^{1t} = e^t$ & $y_2 = e^{2t}$
- ▷ Find the Wronskian, ${y_1}' = e^t$ & ${y_2}' = 2e^{2t}$, therefore:
  $W = \begin{vmatrix} e^t & e^{2t} \\ e^t & 2e^{2t} \end{vmatrix} = e^t \cdot 2e^{2t} - e^t \cdot e^{2t} = e^{3t}$
- ▷ $v_1(t) = \int \frac{-e^{-t}e^{2t}}{e^{3t}}dt = -\int e^{-2t} = \frac{e^{-2t}}{2}$
  $v_1(t) = \int \frac{e^{-t}e^t}{e^{3t}}dt = \int \frac{1}{e^{3t}} = -\frac{e^{-3t}}{3}$
- ▷ There for the general solution is $y_h + h_p = (C_1e^t + C_2e^{2t}) + \left(\frac{e^{-2t}}{2} \cdot e^t - \frac{e^{-3t}}{3} \cdot e^{2t}\right)$
- ▷ $= C_1e^t + C_2e^{2t} + \frac{e^{-t}}{6}$

Variable Coefficient Equations:
The Cauchy-Euler or equidimensional equation: $at^2(t)y'' + bt(t)y' + c(t)y = f(t)$

- Apply the following substitutions, $y = t^r,\ ty' = trt^{r-1} = rt^r,\ t^2y'' = t^2r(r-1)t^{r-2} = r(r-1)t^r$
- Simplify to, $ar^2 + (b-a)r + c = 0$ aka the characteristic equation.
- Solve using convectional techniques

NOT DONE

Basic Theory of Linear Differential Equations:
Verify that a solution exists and is unique for the IVP
For the variable coefficient equation: $a_2(t)y'' + a_1(t)y' + a_0(t)y = f(t)$
Aka the Cauchy-Euler or equidimensional equation: $at^2(t)y'' + bt(t)y' + c(t)y = f(t)$

- Divide both sides by $a_2(t)$ making $y''(t) + p(t)y'(t) + q(t)y(t) = g(t)$
- Find the interval $I = (a, b)$ in which the function is continuous.
- Find the sub interval in which a given point $y(x_0) = Y_0$ and $y'(x_0) = {Y_0}'$ exists.

Homogeneous Linear Equations with Constant Coefficients:
$*$ Refer to 4.2 Homogeneous Linear ODE of 2^{nd} Order of Unit Guide 2.

Undetermined Coefficients and the Annihilator Method:
The annihilator of a function is a differential operator which, when operated on it, obliterates it. The annihilator of $f(x)$ when multiplied against $f(x)$ will equal 0. Annihilators can be used to make non-homogeneous functions homogeneous.

To find an annihilator for a function:

- For an auxiliary equation rewrite $ay'' + by' + cy = f(x)$ as $Ł[y] = (aD^2 + bD + c)[y] = f(x)$, Factor to attain the annihilator $Ł[y]$.
- For an equation $f(x) = f_1 + f_2 + ... + f_n$
  - Create a table for each f_n:
    $\begin{cases} f_n \\ Df_n \\ D^2 f_n \\ ... \end{cases}.$
  - For two or more items from the table, set equal to zero by adding together and multiplying respective factors: $aD_1 f_n + aD_2 f_n + ... = (aD_1 + bD_2 + ...)[f]$
  - Factor $aD_1 + bD_2 + ...$ to attain the annihilator.
  - Annihilators for $f_1 + f_2 + ... + f_n$ will multiply together.

ex.

- ▷ Find the annihilator for $y'' + 25y = 6\sin(x)$
- ▷ Rewrite $y'' + 25y$ as $(1D^2 + 25)[y]$
- ▷ Factor, $D^2 + 25 = (D + 5i)(D - 5i)$

▷ Create a table for $6\sin(x)$:
$$\begin{cases} f(x) = 6\sin(x) \\ Df(x) = 6\cos(x) \\ D^2 f(x) = -6\sin(x) \end{cases}$$

▷ Set two or more terms equal to zero, $af(x) + bD^2f(x) = 6\sin(x) - 6\sin(x) = 0$, $a = 1$ & $b = 1$

▷ Factor, $f(x) + D^2f(x) = (D^2+1)[f(x)] = (D+i)(D-i)[f(x)]$

▷ The annihilators for the equation are:
$(D+5i)(D-5i) = (D+i)(D-i)$

$f(x)$	Annihilator
$a_0 + a_1x + a_2x^2 + ... + a_nx^n$	D^{n+1}
e^{rx}	$D - r$
x^ne^{rx}	$D - r$^{n+1}
$\cos(bx)$ or $\sin(bx)$	$D^2 + b^2$
$x^n\cos(bx)$ or $x^n\sin(bx)$	$(D^2 + b^2)^{n+1}$
$e^{ax}\cos(bx)$ or $e^{ax}\sin bx$	{(D-a)^2+b^2=D^2-2aD+a^2+b^2}
$x^ne^{ax}\cos(bx)$ or $x^ne^{ax}\sin(bx)$	$((D-a)^2 + b^2)^{n+1} = (D^2 - 2aD + a^2 + b^2)^{n+1}$

To use the annihilator method to convert the non-homogeneous equation to a homogeneous equation aka finding the operator:

- Find the annihilators for $ay'' + by' + cy = f(x)$
- Substitute the left annihilators in, $(left\ annihilators)[y] = f(x)$
- Multiply both sides by the right annihilators to make $f(x) = 0$,
$(right\ annihilators)(left\ annihilators)[y] = 0$
- (To solve use the method of undetermined coefficients)

ex.

▷ Convert the function $y'' - 12y' + 32y = 4\cos(3x)$ to a homogeneous equation

▷ Substitute the left annihilator, $(D^2 - 12D + 32)[y] = 4\cos(3x)$

▷ Multiply by the right annihilator, $(D^2+9)(D^2 - 12D + 32)[y] = (D^2+9)(4\cos(3x))$

▷ Therefore the operator equals $(D^2+9)(D^2 - 12D + 32)[y] = 0$

Method of Undetermined Coefficients:
∗ Refer to 4.4 Method of Undetermined Coefficients in Unit Guide 2.

Method of Variation of Parameters:
For the auxillary equation $y^n + p(t)y^{n-1} + ... + q(t)y = g(t)$

If $f(t) \neq 0$ aka non-homogeneous

- Find $y_1(t), y_2(t), ..., y_n(t)$ via the homogeneous solution $y_h(t) = C_1y_1(t)+C_2y_2(t)+...+C_ny_n(t)$.
- Calculate the Wronskian $W = \begin{vmatrix} y_1 & y_2 & y_3 & ... \\ {y_1}' & {y_2}' & {y_3}' & ... \\ {y_1}'' & {y_2}'' & {y_3}'' & ... \\ ... & ... & ... & ... \end{vmatrix} = ...$
- Calculate the Wronskians $W(y_m)$,
$W(y_1) = \begin{vmatrix} 0 & y_2 & y_3 & ... \\ 0 & {y_2}' & {y_3}' & ... \\ 0 & {y_2}'' & {y_3}'' & ... \\ f(t) & ... & ... & ... \end{vmatrix}, W(y_2) = \begin{vmatrix} y_1 & 0 & y_3 & ... \\ {y_1}' & 0 & {y_3}' & ... \\ {y_1}'' & 0 & {y_3}'' & ... \\ ... & f(t) & ... & ... \end{vmatrix}, etc...$
- Find the particular solution $y_p(t) = \sum_{m=1}^{n} y_m \int \frac{g(t)W(y_m)}{W(y_1,y_2,...,y_n)}dx$
- The general solution is $y_h + y_p$.
- For an initial value problem, solve for C_1, C_2, ...,C_n, by plugging, the general solution $y_h + y_p$ into the auxiliary equation at the respective derivatives.

ex.

▷ Find the general solution of $y''' - 2y'' + y' = x$

▷ Substitute $r^3 - 2r^2 + r$ for $y''' - 2y'' + y' = x$ to find homogeneous solution (y_h)

▷ $r(r-1)^2$, therefore roots are 0 & 1

▷ Homogeneous solution (y_h) $= C_1e^{0t} + C_2e^{1t} + C_3te^{1t}$, therefore $y_1 = 1$, $y_2 = e^t$, & $y_3 = te^t$.

▷ Find the Wronskian, $W(y_1, y_2, y_3) = \begin{vmatrix} 1 & e^t & te^t \\ 0 & e^t & te^t + e^t \\ 0 & e^t & te^t + 2e^t \end{vmatrix} = e^{2t}$

▷ Find the Wronskians, $W(y_m)$,
$W(y_1) = \begin{vmatrix} 0 & e^t & te^t \\ 0 & e^t & te^t + e^t \\ 1 & e^t & te^t + 2e^t \end{vmatrix} = e^{2t}$, W(y$_2$) $= \begin{vmatrix} 1 & 0 & te^t \\ 0 & 0 & te^t + e^t \\ 0 & 1 & te^t + 2e^t \end{vmatrix} = e^t - te^t$,
& $W(y_3) = \begin{vmatrix} 1 & e^t & 0 \\ 0 & e^t & 0 \\ 0 & e^t & 1 \end{vmatrix} = e^t$

- ▷ Solve for each of the items in the particular solution
- ▷ Therefore the general solution is,
  $y_h + h_p = (C_1 + C_2 e^t + C_3 te^t) + \left(1 \int \frac{te^{2t}}{e^{2t}} dx \ - \ e^t \int \frac{t(e^t + te^t)}{e^{2t}} dx \ + \ te^t \int \frac{te^t}{e^{2t}} dx\right)$