**Formulas:**

- Galilean Transformation
  - $x' = x - vt$ (Length)
  - $t' = t$ (Time)
  - $u' = u - v$ (Velocity)
- Requirement of a linear trajectory within a frame,
  $x = ct$ (Proper) & $x' = ct'$ (Relative)
- Lorentz Transformation (Measured at different length/time)
  - Length
    * $x' = \gamma(x - \beta ct)$ (Have Proper, Find Relative)
    * $x = \gamma(x' + \beta ct')$ (Have Relative, Find Proper)
  - Time
    * $ct' = \gamma(ct - \beta x)$ (Have Proper, Find Relative)
    * $ct = \gamma(ct' + \beta x')$ (Have Relative, Find Proper)
  - Velocity Addition/Subtraction
    * $u' = \frac{u-v}{1-\frac{uv}{c^2}}$ (Have both Proper, Find Relative)
    * $u = \frac{u'+v}{1+\frac{u'v}{c^2}}$ (Have a Relative & a Proper, Find Proper)
  - Relativistic Factor, $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$
  - Fraction of Speed of Light, $\beta = \frac{v}{c}$
- Dilation/Contraction (Measured at same length/time)
  - Length, $L = \gamma L_0$ (Have Proper ($L_0$), Find Relative ($L$)
  - Time, $t = \gamma t_0$ (Have Proper ($t_0$), Find Relative ($t$)
- Simultaneity, events are simultaneous if an observer measures them occurring at the same time.
- Energy
  - Rest Energy, $E_0 = mc^2$
  - Kinetic Energy, $K = (\gamma_u - 1)mc^2$
  - Momentum, $\vec{p} = \gamma_u m\vec{u}$
  - Total Energy, $E = K + E_0 \equiv \gamma_u mc^2 = (\gamma_u - 1)mc^2 + mc^2$

**Solving Time Problems**:
ex. Suppose we (on Earth) observe a Solar flare that lasts for 3.5 minutes. According to clocks on a spaceship that is traveling at 0.4c relative to our Solar system (in the direction from the Sun towards Earth), how many minutes does the flare last? *Since the solar flare is an event taking place at a single location we utilize the time dilation formula, and since the observer is at rest relative to the sun we solve for the relative time (t).*
ex. In a frame that is at rest with our Sun, light takes 8.0 minutes to reach Earth. According to a clock placed on a spaceship travelling at 0.40c relative to the Sun (in the direction from the Sun towards Earth) how much time does it take for the light to travel from the Sun to Earth? *Since we are measuring the time/distance for light to travel we use the lorentz transformation, and the time we have is from an observer at rest relative to the sun, so we solve for the relative time.*
ex. I stand midway between two rockets as they are about to launch. According to my clock, the rocket to my right launches 0.4d/c seconds before the one on the left, where d is the distance between me and each

of the rockets, and c is the speed of light in vacuum. Is there an inertial frame in which the two launches are simultaneous? *Yes, set two Lorentz transformations equal to each other, for Rocket*$_R$ $t = 0$ *&* $x = -d$ *for the other Rocket*$_L$ $t = .6\frac{d}{c}$ *&* $x = d$*, and solve for , remember all the possible substitutions*

**Solving Velocity Addition/Subtraction Problems**:

ex. In a space race, two ships are traveling at 0.48c and 0.54c, respectively. How fast are the two ships traveling relative to each other? (report the magnitude of the velocity, in fraction of the speed of light.) *Solve for the relative as we have both their proper Velocities.*

ex. A spaceship traveling at 0.20c relative to our solar system, releases a stream of protons at 0.802c relative to the spaceship in the same direction that it is traveling. What is the speed of the protons relative to our solar system? (report your answer as a fraction of the speed of light.) *We have one proper and the relative so solve for the other proper.*

**Solving Energy Problems**:

ex. Suppose a particle accelerator accelerates a proton to the point that its total relativistic energy in the lab frame is 4.12 times its rest energy – what is its speed in the lab frame? (report your answer as a fraction of the speed of light, to 3 decimal places.) $E = \gamma_u mc^2$, $E_0 = mc^2$ *therefore* $\gamma = 4.12$*, then solve for* $\beta$

ex. What is the relativistic momentum of an electron in a frame in which its kinetic energy is five times its rest energy? $K = (\gamma_u - 1)E_0$ *so* $\gamma_u = 6$*, solve for speed* $\vec{u}$*, plug into momentum.*

**Space Time Diagrams:**