Signal Analysis & Analog Communication

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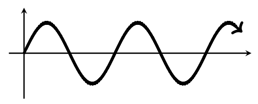
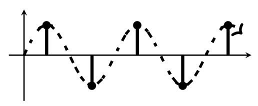
——— ITEMS ARE NOT IN ORDER ———

Analog v. Digital

Analog signals have a continuous range of amplitude, while digital signals have discrete amplitude values.

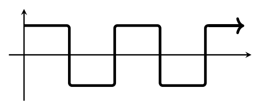


Analog can be converted to digital via quantization, the process of cutting a signal at a certain period and rounding to the nearest discrete value.



Finite energy $\Rightarrow$ Energy Signal

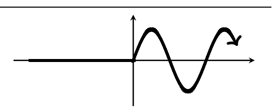
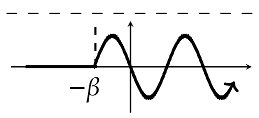
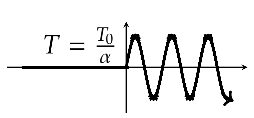
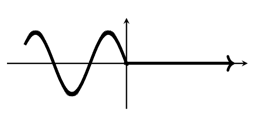
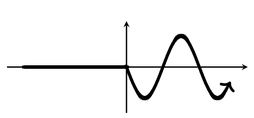
Finite & non-zero power $\Rightarrow$ Power Signal



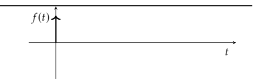
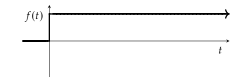
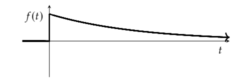
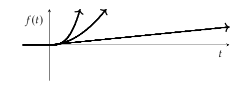
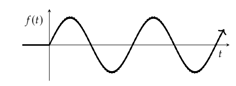
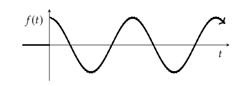
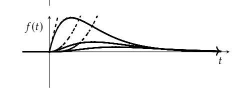
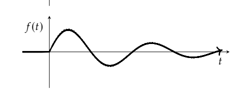
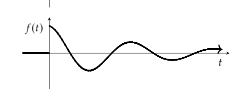
Signal Classification

Classification	Definition
Continuous-Time	Defined for continuous range of time.
Discrete-Time	Defined for discrete range of time.
Analog	Amplitude can take infinite number of values.
Digital	Amplitude can take finite number of values.
Periodic	Satisfies $g(t) = g(t + T_0)$ for all t, aka repeats.
Aperiodic	Does not repeat.
Energy	Has finite energy.
Power	Has finite/non-zero power.
Deterministic	Described via mathematics.
Probabilistic	Described as random/noise.

Signal Operations

Operator	Expression	Graph
Original	$g(t)$	
Time Shifting	$g(t) \rightarrow g(t+\beta)$	
Time Scaling	$g(t) \rightarrow g(\alpha t)$	
Time Inversion	$g(t) \rightarrow g(-t)$	
Amplitude Inversion	$g(t) \rightarrow -g(t)$	

Laplace Transforms

Type	$f(t)(t < 0-)$	$F(s) = \mathscr{L} f(t)$	Graph
Impulse	$\delta(t)$	1	
Step	$u(t)$	$\frac{1}{s}, s > 0$	
Exponential	e^{-at}	$\frac{1}{s+a}, s > a$	
Ramp	t^n	$\frac{n!}{s^{n+1}}, s > 0$	
Sine	$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}, s > 0$	
Cosine	$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}, s > 0$	
Damped Ramp	$e^{-at}t^n$	$\frac{n!}{(s-a)^{n+1}}, s > a$	
Damped Sine	$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}, s > a$	
Damped Cosine	$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}, s > a$	

Signal Characteristics

Channel Bandwidth (B) is the range of frequencies that can transmit with good quality. The SNR or Signal to Noise Ratio describes how easy it is to recover a signal.

$$\text{SNR} = \frac{\text{Signal Power}}{\text{Noise Power}}$$

High SNR means more signal levels and thus more bits per pulse, whereas higher bandwidth means more pulses per second.

This culminates in Channel Capacity (C) which is the maximum number of binary bits that can be transmitted in a second.

$$\text{C} = \text{B} \cdot \log_2(1 + \text{SNR}) \text{ bit/sec}$$

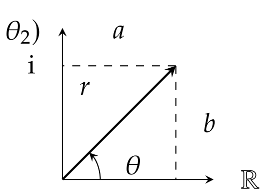
(For a channel with additive Gaussian white noise.)

Complex Numbers

$$j = \sqrt{-1}$$

Form	Equation	Conversion
Rectangular	$a + jb$	To polar: $r = \sqrt{a^2+b^2}, \angle\theta = \tan^{-1}(\frac{b}{a})$
Polar	$re^{j\theta} = r\angle\theta$	To rectangular: $a = rcos(\theta), b = rsin(\theta)$

Operation	Property
Addition	$(a_1 + jb_1) + (a_2 + jb_2) = (a_1 + a_2) + j(b_1 + b_2)$
Subtraction	$(a_1 + jb_1) - (a_2 + jb_2) = (a_1 - a_2) + j(b_1 - b_2)$
Multiplication	$r_1\angle\theta_1 \cdot r_2\angle\theta_2 = (r_1 \cdot r_2)\angle(\theta_1 + \theta_2)$
Division	$\frac{r_1\angle\theta_1}{r_2\angle\theta_2} = \frac{r_1}{r_2}\angle(\theta_1 - \theta_2)$



deg	0°	30°	45°	60°	90°
rad	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$$

$1 + j = \sqrt{2}\angle 45°$

$\{a + jb\}^* = a - jb$

$\frac{1}{j} = -j$

Euler's Formula: $e^{j\theta} = \cos\theta + j\sin\theta$

Sifting Property of the Impulse Function

The Dirac/delta/impulse function is defined by: $\int_{-\infty}^{\infty} \delta(t)dt = 1$, this states that there is a vertical line with height infinity whose encompassing area equals one. The sift property creates a infinite vertical line at the point $(t, 0)$ whose area is defined by the function $\phi(T)$. This allows the extraction of a point value.

$$\int_{-\infty}^{\infty} \phi(t)\delta(t-T)dt = \phi(T)$$

Signals as Vectors

Signals can be considered as vectors in a mathematical space, represented by their magnitude and phase. These signals can be decomposed into orthogonal components, such as $\cos(x)$ and $\sin(x)$, or $e^{j\theta}$ and $e^{-j\theta}$.

For a trigonometric Fourier series, the vector representation would be:

$$\langle a_0, a_1, b_1, \ldots, a_n, b_n \rangle$$

Where a_0 represents the DC component, a_n are the coefficients of the cosine terms, and b_n are the coefficients of the sine terms.

Conversely, for the exponential Fourier series, the representation would be:

$$\langle D_{-n}, \cdots, D_{-1}, D_0, D_1, \cdots, D_n \rangle$$

Where D_n represents the complex Fourier coefficients corresponding to both positive and negative frequencies, including the DC component D_0.

Vector Operations

Magnitude of a vector: $||\vec{a}|| = \sqrt{a_1^2 + a_2^2 + \cdots + a_n^2}$

Angle between vectors: $\vec{a} \cdot \vec{b} = ||\vec{a}||||\vec{b}|| \cos\theta$ & $\vec{a} \times \vec{b} = ||\vec{a}||||\vec{b}|| \sin\theta\ \hat{n}$

Unit/Normalized Vector: $\hat{a} = \frac{\vec{a}}{||a||}$

Inner/Dot Product (if 0 then vectors are orthogonal): $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + \cdots + a_nb_n$

Cross Product (if 0 then vectors are parallel):

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \vec{b}_x & \vec{b}_y & \vec{b}_z \end{vmatrix} = i\left[\vec{a}_y\vec{b}_z - \vec{a}_z\vec{b}_y\right] - j\left[\vec{a}_x\vec{b}_z - \vec{a}_z\vec{b}_x\right] + k\left[\vec{a}_x\vec{b}_y - \vec{a}_y\vec{b}_x\right]$$

Scaler Projection: $\text{comp}_{\vec{a}}\,\vec{b} = \frac{\vec{a} \cdot \vec{b}}{||\vec{a}||}$ Vector Projection: $\text{proj}_{\vec{a}}\,\vec{b} = \frac{\vec{a} \cdot \vec{b}}{||\vec{a}||} * \frac{\vec{a}}{||\vec{a}||}$

Signal Energy & Power

Energy $= E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt$ Power $= P_g = \frac{1}{T}\int_T |g(t)|^2 dt$

$P_{\text{dBW}} = 10 \cdot \log_{10} P$ $P_{\text{dBm}} = 30 + P_{\text{dBW}}$

Signal Approximation & Correlation

To approximate a signal $g(t)$ in terms of another $x(t)$, apply:

$$g(t) \cong cx(t), t \cong [t_1, t_2], \quad \text{where } c = \frac{1}{E_x}\int_{t_1}^{t_2} g(t)x(t)dt = \frac{\langle g, x\rangle}{\langle x, x\rangle}$$

$$\implies \text{Error} = e(t) = g(t) - cx(t), t \in [t_2, t_1] \;\;\&\;\; \text{Error Energy} = E_e = \int_{t_1}^{t_2} |e(t)|^2 dt$$

Measure of Similarity (c) describes how well a signal can replicate another.

	Description	Equation
Correlation	Relationship between two signals.	$\rho = \frac{1}{\sqrt{E_g E_z}}\int_{t_1}^{t_2} z(t)g^*(t)dt = \frac{\langle g,x\rangle}{\lVert g\rVert \cdot \lVert x\rVert}$
Cross-Correlation	Similarity between two different signals over time.	$\psi_{zg}(\tau) = \int_{-\infty}^{\infty} z(t)g^*(t-\tau)dt$
Auto-Correlation	Similarity of a signal with itself over time.	$\psi_g(\tau) = \int_{-\infty}^{\infty} g(t)g(t+\tau)dt$

$$\begin{matrix}-1 \le \rho \le 1 \\ -1 \le \cos\theta \le 1\end{matrix} \longrightarrow \begin{matrix}\rho = -1 \\ \theta = \pi\end{matrix}\text{ Opposite}, \;\; \begin{matrix}\rho = 0 \\ \theta = \frac{\pi}{2}\end{matrix}\text{ Orthogonal}, \;\; \& \;\; \begin{matrix}\rho = 1 \\ \theta = 0\end{matrix}\text{ Identical}$$

Gram Schmidt Method

Orthogonal signals are signal vectors who are at a right angle or are $\frac{\pi}{2}$ out of phase of one another. $\cos(t)$ & $\sin(t)$ are examples of orthogonal and basis functions.

$$\int_{t_1}^{t_2} x_m(t)x_n^*(t)dt = E_n \begin{matrix} = 0 \implies \text{Orthogonal} \\ = 1 \implies \text{Normalized}\end{matrix}$$

Where $x_m(t)$ & $x_n(t)$ are two separate signals. The conjugate transformation accommodates for complex signals.

To orthogonalize a set of vectors $\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_k$ perform the following operation:

$$\vec{u}_1 = \vec{v}_1 \quad \vec{u}_2 = \vec{v}_2 - \text{proj}_{\vec{u}_1}\vec{v}_2 \quad \vec{u}_3 = \vec{v}_3 - \text{proj}_{\vec{u}_1}\vec{v}_3 - \text{proj}_{\vec{u}_2}\vec{v}_3 \quad \cdots$$

$$\vec{u}_k = v_k - \sum_{j=1}^{k-1} \text{proj}_{u_j} v_k$$

The new set of vectors $\vec{u}_1, \vec{u}_2, \cdots, \vec{u}_k$ are mutually orthogonal and normalized. While the magnitude has changed the resulting summation of vectors still results in the same direction as the summation of the originals.

Trigonometric Fourier Series

The Trigonometric Fourier Series approximates functions through the summation of increasingly higher frequency cos & sin terms. $T_0 = \frac{2\pi}{\omega_0} = 2\pi f$

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t), \quad t_1 \le t \le t_1 + T_0$$

$$a_0 = \frac{1}{T_0}\int_{t_1}^{t_1+T_0} g(t)dt \qquad a_n = \frac{2}{T_0}\int_{t_1}^{t_1+T_0} g(t)\cos(n\omega_0 t)dt$$

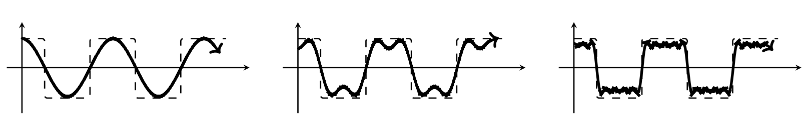
$$b_n = \frac{2}{T_0}\int_{t_1}^{t_1+T_0} g(t)\sin(n\omega_0 t)dt \qquad \text{where } n = 1, 2, 3, \cdots$$

Alternatively a function can be expressed in the Compact Fourier Series form.

$$g(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n), \quad t_1 \le t \le t + T_0$$

$$C_0 = a_0 \qquad C_n = \sqrt{a_n^2 + b_n^2} \qquad \theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right)$$

If odd $g(t) = -g(-t) \rightarrow a_n = 0$ If even $g(t) = g(-t) \rightarrow b_n = 0$



$n = 1$ $n = 3$ $n = 9$

Exponential Fourier Series

By using Euler's Formula the Trigonometric Fourier Series can be described in exponential terms.

$$\underbrace{e^{j\theta} = \cos(\theta) + j\sin(\theta)}_{\text{Euler's Formula}} \rightarrow \begin{matrix}\cos(\theta) = \frac{e^{j\theta}+e^{-j\theta}}{2} \\ \sin(\theta) = \frac{e^{j\theta}-e^{-j\theta}}{2}\end{matrix} \implies \begin{matrix}\cos(n\omega_0 t) \\ \sin(n\omega_0 t)\end{matrix} \longrightarrow \begin{matrix}e^{jn\omega_0 t} \\ e^{-jn\omega_0 t}\end{matrix}$$

$$g(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t} = D_0 + \sum_{n=1}^{\infty} D_n e^{jn\omega_0 t} + D_{-n}e^{-jn\omega_0 t}$$

$$D_0 = C_0 = a_0 \qquad D_n = \frac{1}{T_0}\int_{T_0} g(t)e^{-jn\omega_0 t}dt$$

$$\begin{matrix}D_n = \frac{1}{2}C_n e^{j\theta_n} \\ D_{-n} = \frac{1}{2}C_n e^{-j\theta_n}\end{matrix} \quad \text{where} \quad D_n = D_{-n}^*, \; |D_n| = |D_{-n}|, \;\&\; \angle D_n = -\angle D_{-n}$$

Parseval's Theorem

The energy of a signal in the time domain is equal when expressed in the frequency domain.

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega = \int_{-\infty}^{\infty} |X(2\pi f)|^2 df$$

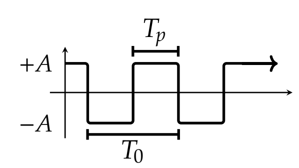
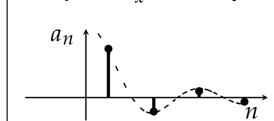
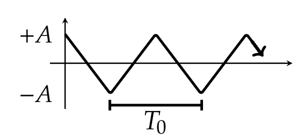
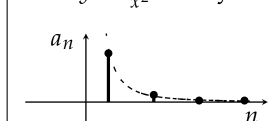
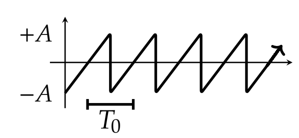
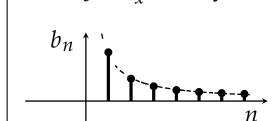
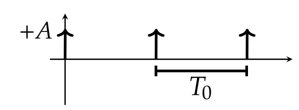
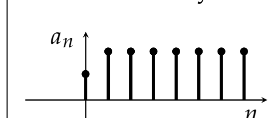
Fourier Spectrum

The Fourier Spectra allows visualization of the amplitude and phase over the course of a Fourier Series function in terms of integer multiples (n) or integer multiples of frequency ($\omega = n \cdot \omega_0$).

$$\text{Consider } C_n \;\&\; \theta_n \text{ or } |D_n| \;\&\; \angle D_n \text{ versus } n.$$

Entirely even ($a_n \neq 0$ & $b_n = 0$) or odd ($a_n = 0$ & $b_n \neq 0$) functions lack the θ_n & $\angle D_n$ factor. It is only functions who are neither even or odd that require both sin & cos components, and therefore θ_n & $\angle D_n$.

Waveform Examples

Waveform	Amplitude Spectra	Coefficients
Square 	$y = \frac{\sin(x)}{x}$ Decay 	$a_0 = 0$ $a_n = \frac{4A\sin\left(n\pi\frac{T_p}{T_0}\right)}{n\pi}$ $b_n = 0$
Triangular 	$y = \frac{1}{x^2}$ Decay 	$a_0 = 0$ $a_n = \frac{8A\sin\left(\frac{n\pi}{2}\right)^2}{n^2\pi^2}$ $b_n = 0$
Sawtooth 	$y = \frac{1}{x}$ Decay 	$a_0 = 0$ $a_n = 0$ $b_n = \frac{2A}{n\pi}$
Impulse Train 	No Decay 	$a_0 = \frac{A}{T_0}$ $a_n = \frac{2A}{T_0}$ $b_n = 0$

Signal Operations

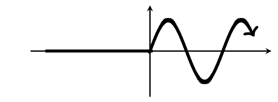
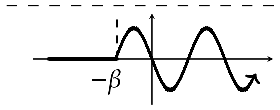
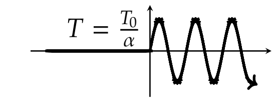
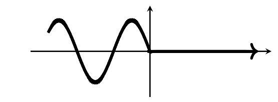
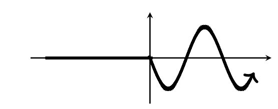
Operator	Expression	Graph
Original	$g(t)$	
Time Shifting	$g(t) \rightarrow g(t+\beta)$	
Time Scaling	$g(t) \rightarrow g(\alpha t)$	
Time Inversion	$g(t) \rightarrow g(-t)$	
Amplitude Inversion	$g(t) \rightarrow -g(t)$	

Table of Fourier Transforms

g(t)	G(f)	Condition
$e^{-at}u(t)$	$\frac{1}{a+j2\pi f}$	$a > 0$
$e^{at}u(-t)$	$\frac{1}{a-j2\pi f}$	$a > 0$
$e^{-a\lvert t\rvert}$	$\frac{2a}{a^2+(2\pi f)^2}$	$a > 0$
$te^{-at}u(t)$	$\frac{1}{(a+j2\pi f)^2}$	$a > 0$
$t^n e^{-at}u(t)$	$\frac{n!}{(a+j2\pi f)^{n+1}}$	$a > 0$
$\delta(t)$	1	$\cdots$
1	$\delta(f)$	$\cdots$
$e^{j2\pi f_0 t}$	$\delta(f - f_0)$	$\cdots$
$u(t)$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$	$\cdots$
sgn(t)	$\frac{2}{j2\pi f}$	$\cdots$
$\cos(2\pi f_0 t)u(t)$	$\frac{1}{4}\left[\delta(f-f_0) + \delta(f+f_0)\right] + \frac{j2\pi f}{(2\pi f_0)^2-(2\pi f)^2}$	$\cdots$
$\sin(2\pi f_0 t)u(t)$	$\frac{1}{4j}\left[\delta(f-f_0) - \delta(f+f_0)\right] + \frac{2\pi f_0}{(2\pi f_0)^2-(2\pi f)^2}$	$\cdots$
$e^{-at}\sin(2\pi f_0 t)u(t)$	$\frac{2\pi f_0}{(a+j2\pi f)^2+4\pi^2 f_0^2}$	$a > 0$
$e^{-at}\cos(2\pi f_0 t)u(t)$	$\frac{a+j2\pi f}{(a+j2\pi f)^2+4\pi^2 f_0^2}$	$a > 0$
$\Pi\left(\frac{t}{\tau}\right)$	$\tau\text{sinc}(\pi f\tau)$	$\cdots$
$2B\text{sinc}(2\pi Bt)$	$\Pi\left(\frac{f}{2B}\right)$	$\cdots$
$\Delta\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2}\text{sinc}^2\left(\frac{\pi f\tau}{2}\right)$	$\cdots$
$B\text{sinc}^2(\pi Bt)$	$\Delta\left(\frac{f}{2B}\right)$	$\cdots$
$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$f_0\sum_{n=-\infty}^{\infty}\delta(f-nf_0)$	$f_0 = \frac{1}{T}$
$e^{\frac{-t^2}{2\sigma^2}}$	$\sigma\sqrt{2\pi}e^{-2(\sigma\pi f)^2}$	$\cdots$

Properties of Fourier Transforms

Property	Transform Pair/Property
Superposition	$g_1(t) + g_2(t) \leftrightarrow G_1(f) + G_2(f)$
Scalar Multiplication	$kg(t) \leftrightarrow kG(f)$
Duality	$G(t) \leftrightarrow g(-f)$
Time scaling	$g(at) \leftrightarrow \frac{1}{\lvert a\rvert} G\left(\frac{f}{a}\right)$
Time Shifting	$g(t - t_0) \leftrightarrow G(f)e^{-j2\pi f t_0}$
Frequency Shifting	$g(t)e^{j2\pi f_0 t} \leftrightarrow G(f - f_0)$
Time Convolution	$g_1(t) * g_2(t) \leftrightarrow G_1(f)G_2(f)$
Frequency Convolution	$g_1(t)g_2(t) \leftrightarrow G_1(f) * G_2(f)$
Modulation via Sine	$g(t)\sin(2\pi f_0 t) \leftrightarrow \frac{1}{2j}\left[G(f - f_0) - G(f + f_0)\right]$
Modulation via Cosine	$g(t)\cos(2\pi f_0 t) \leftrightarrow \frac{1}{2}\left[G(f - f_0) + G(f + f_0)\right]$
Time Differentiation	$\frac{d^n g(t)}{dt^n} \leftrightarrow G(f)(j2\pi f)^n$
Time Integration	$\int_{-\infty}^{t} g(x)dx \leftrightarrow \frac{G(f)}{j2\pi f} + \frac{1}{2}G(0)\delta(f)$

Signal Distortion

There are four forms of distortion: linear, nonlinear, multi-path, & fading.

Distortion-less transforms include gain or time delay.
Where $|H(f)| = k$ & $\theta_h(f) = -2\pi f t_d$

$$\text{Group Delay: } t_d(f) = -\frac{1}{2\pi}\frac{d\theta_h(f)}{df}$$

Linear distortions do not have a constant gain nor do they maintain a linear trend of phase. Where $|H(f)| \neq k$ & $\theta_h(f) \neq -2\pi f t_d$

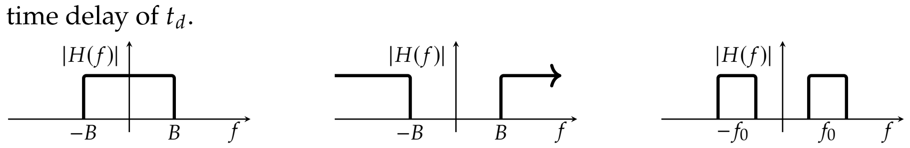
Nonlinear distortions– distortions who do not obey typical trends.

Multipath distortions are where a signal arrives at the receiver through multiple paths of different delays. Both magnitude and phase are periodic with respect to frequency.

Fading is where the channel characteristics change overtime, this may be frequency dependent.

Ideal Filters

Ideal filters have a phase of $\theta_h(f) = -2\pi f t_d$, aka a linear phase which results in a time delay of t_d.



Low-pass High-pass Band-pass

N^{th} Order Low-pass $|H(f)| = \frac{1}{\sqrt{1+\left(\frac{f}{B}\right)^{2N}}}$ 3dB $= \frac{1}{\sqrt{2}} = 0.707$

Power & Energy Spectral Density

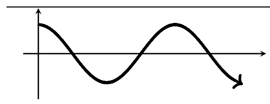
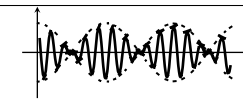
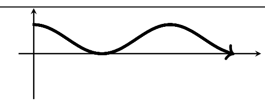
Energy: The essential bandwidth (B) will contain 90%, 95%, or 99% of the total energy depending on the application.

$$E_g = \int_{-\infty}^{\infty} |G(f)|^2 df = \int_{-\infty}^{\infty} \Psi_g(f)df, \quad \text{where } \Psi_g(f) := |G(f)|^2$$

Power: $S_g(f) = \lim_{T\to\infty} \frac{|G_T(f)|^2}{T}$ $\quad P_g = \int_{-\infty}^{\infty} S_g(f)df$

Amplitude Modulation

Message : m(t) Carrier : $\cos(2\pi f_c t)$ Modulated : $s(t) = km(t)\cos(2\pi f_c t)$

Original	Modulated	Demodulated
		
Message being sent	Multiply message by carrier, forms envelope	Multiply again, remove high frequency

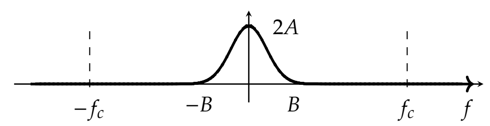
Methods to demodulate:
• Synchronous Detection, where a carrier exactly matching the original is used to demodulate. Must be used if signal is overmodulated i.e. $\mu > 1$.
• Envelope Detection, send a carrier along with modulated signal, $[A + m(t)]\cos(\ldots)$, an envelope exists if $A + m(t) \geq 0$ for all t. Must be $0 \leq \mu \leq 1$.

$$\varphi_{AM} = \underbrace{A\cos(\cdots)}_{\text{carrier}} + \underbrace{m(t)\cos(\cdots)}_{\text{sidebands}} \quad A \geq m_{\text{peak}}(t) \quad \text{Modulation Index } \mu = \frac{m_p}{A}$$

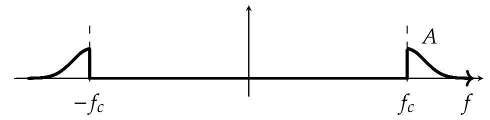
$$P_c = \frac{A^2}{2} \quad P_s = \frac{m^2(t)}{2} \quad \text{Power Efficiency } \eta = \frac{\textbf{Useful Power}}{\textbf{Total Power}} = \frac{P_s}{P_c + P_s}$$

Band — **Spectra**

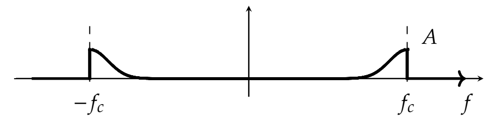
Baseband



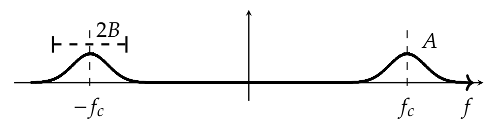
Single Sideband Upper (USB)



Single Sideband Lower (LSB)



Double Sideband (DSB)



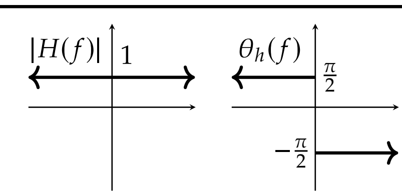
All of the above examples are suppressed carrier (-SC).

Quadrature Amplitude Modulation (QAM), is where two AM signals are combined to double the data rate. The upper channel is the in-phase channel, while the lower channel is the quadrature channel.

$$\varphi_{QAM}(t) = m_1(t)\cos(2\pi f_c t) + m_2(t)\sin(2\pi f_c t)$$

Hilbert Transform

The Hilbert Transform is an ideal phase shifter, that shifts the phase of positive spectral components by $-\frac{\pi}{2}$. $H(f) = -j \cdot \text{sgn}(f)$



Frequency Modulation

Message : $m(t)$ Carrier Frequency : f_c

$$m(t) \rightarrow \boxed{FM} \rightarrow \varphi_{FM}(t) = A\cos\left[2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(t)dt\right]$$

Due to frequency modulation's constant amplitude it registers 30dB stronger. As a result FM signals are more resistant to noise. While AM is linear, FM is non-linear in nature.

Inst. Frequency $f_i(t) = 2\pi f_c + 2\pi k_f m(t)$ Frequency Deviation $\Delta F = \frac{k_f m_p}{2\pi}$
Deviation Ratio $\beta = \frac{\Delta F}{B}$ Carson's Rule $B_{FM} = 2B(\beta + 1)$Hz Power $P_{FM} = \frac{A^2}{2}$

Phase Modulation

PM and FM signals are interchangeable, replacing $m(t)$ with $\int m(t)dt$ converts FM to PM.

$$m(t) \rightarrow \boxed{PM} \rightarrow \varphi_{PM}(t) = A\cos(2\pi f_c t + 2\pi k_p m(t))$$

Inst. Frequency $f_i(t) = 2\pi f_c + 2\pi k_p \frac{dm(t)}{dt}$ Frequency Deviation $\Delta F = \frac{k_p \dot{m}_p}{2\pi}$
All other values match FM.

Useful Integrals

Pretend there is a $+C$ attached to the end of each of the below.

- $\int x^n dx = \frac{x^{n+1}}{n+1}$
- $\int dx = x$
- $\int \cos(x)dx = \sin(x)$
- $\int \sin(x)dx = -\cos(x)$
- $\int \sec^2(x)dx = \tan(x)$
- $\int \csc^2(x)dx = -\cot(x)$
- $\int \sec(x)\tan(x)dx = \sec(x)$
- $\int \csc(x)\cot(x)dx = -\csc(x)$
- $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}(x)$
- $\int -\frac{dx}{\sqrt{1-x^2}} = \cos^{-1}(x)$
- $\int \frac{dx}{1+x^2} = \tan^{-1}(x)$
- $\int -\frac{dx}{1+x^2} = \cot^{-1}(x)$
- $\int \frac{dx}{|x|\sqrt{x^2-1}} = \sec^{-1}(x)$
- $\int \frac{dx}{-|x|\sqrt{x^2-1}} = \csc^{-1}(x)$
- $\int e^x dx = e^x$
- $\int \frac{dx}{x} = \log|x|$
- $\int a^x dx = \frac{a^x}{\log(a)}$
- $\int \cosh(x)dx = \sinh(x)$
- $\int \sinh(x)dx = \cosh(x)$
- $\int \tan(x)dx = -\log|\cos(x)|$
- $\int \cot(x)dx = \log|sin(x)|$
- $\int \sec(x)dx = \log|\sec(x) + \tan(x)|$
- $\int \csc(x)dx = -\log|\csc(x) + \cot(x)|$
- $\int \log(x) = x\log(x) - x$

Integration by Parts

$$\int u \cdot dv = u \cdot v - \int v \cdot du + C$$

Reverse Chain Rule

$$\int f(g(x)) \cdot g'(x)dx = F(g(x)) + C$$

♡ Made by Kiva M. ♡ Check out more of my work at kivamccr.xyz ♡